

The Economic Impact of Federal Rental Housing Program 1989

Final Report

**The Economic Impact of
Federal Rental Housing Program**

by

**George Fallis
Arthur Hosios
Gregory Jump
James E. Pesando
Lawrence B. Smith**

Institute for Policy Analysis

University of Toronto

May 16, 1989

CONTENTS

I. Introduction	1
II. Principal Findings	4
III. Auxiliary Findings	10

Summary Tables 1 through 4

Appendix A: Calculating the Present Value of Rental Subsidies

Appendix B: A Framework for the Analysis of Rental Housing Programs

Appendix C: An Econometric Model of the Residential Housing Market in Canada

Appendix D: Simulations of the Macroeconomic Impacts of the CMHC Rental Subsidy Programs on the Canadian Economy: 1971-1987

I. Introduction

1. The purpose of this Report is to assist the CMHC in its evaluation of four former rental housing stimulation plans: the Limited Dividend Program (LD), the Assisted Rental Program (ARP), the Multiple Unit Residential Building Program (MURB), and the Canada Rental Supply Plan (CRSP).
2. The focus of our Report is on the economic impact of these federal programs. We assess their impact on starts and the stock of multiple housing, as well as rents, vacancy rates, and the price and stock of single housing. We determine the impact of these programs on the aggregate level of output and employment, and their cyclical variability, as well as on federal and provincial surpluses or deficits.

The details of the analysis are contained in four Appendixes.

- (1) In Appendix A, we calculate the present value of the subsidies delivered by each of the federal programs for each quarter in the period 1970 to 1987. These summary measures of profitability allow us to account for the fact that the programs have multi-year impacts on each unit, and entail very different subsidy restriction packages (interest subsidy vs. loan/grant vs. tax benefits). Among its many advantages, this approach allows us to "stack" federal and provincial programs in an appropriate way. In this Appendix, we also include a

detailed discussion of the problem of identifying program costs and allocating these costs over time.

- (2) In Appendix B, we provide an economic overview of how the federal programs will affect the market for rental units. We devote particular attention to the role of builders' expectations about the future level of rents, the fact that temporary rental initiatives cannot permanently increase the stock of rental housing, and the factors which affect the rapidity with which rental starts first expand and then decline in response to market forces.
- (3) In Appendix C, we estimate a quarterly model of the market for single and multiple housing units. The capitalized values of the subsidies associated with MURB, and with LD, ARP and CRSP, appear as explanatory variables in the equation designed to explain multiple starts. By effectively increasing the price received by builders/investors for multiple units, the federal programs provide a stimulus to starts.
- (4) In Appendix D, we assess the macroeconomic impact of the federal initiatives. We identify their impact on employment, output, government revenues and expenditures. To do so, we embed our model of the housing market into the FOCUS macroeconometric model of the Canadian economy. FOCUS is a large-scale, quarterly model maintained at the Institute for

Policy Analysis and is designed for policy simulation and assessment.

3. In the text of our report, we summarize the principal findings. We then draw attention to a number of methodological or auxiliary issues that we believe to be of particular interest.
4. The stock-flow model which underlies our analysis operates as follows. There is, at each point in time, a stock demand for rental housing, which depends on real incomes, the real level of rents, and demographic factors. Following the introduction of a temporary federal initiative (such as MURB), builders/entrepreneurs find it profitable to add to the stock of rental units, and additional starts occur. The subsequent increase in the stock of rental units exerts downward pressure on rents, and thus operates to reduce starts in subsequent periods. Once the temporary initiative is withdrawn, the profitability of new rental units declines relative to the pre-initiative level, since rents have declined relative to what they would otherwise be. As discussed in Appendix B, long-run equilibrium will occur, other things equal, when the stock of multiples settles at its pre-initiative level. (This abstracts from population, household and real income growth.) If rent controls bind, so that excess demand for rental units exist, the initial increase in the stock of rental units may not serve to reduce rents. In this situation, a temporary program will have a longer-lived impact on the stock of rental housing but still not have a permanent effect because of the eventual depreciation of any program-induced additions to that

stock. To the extent that rent controls serve only to slow the adjustment of rents, as in our econometric model, the stock adjustment process proceeds more rapidly.

II. Principal Findings

1. Our econometric results confirm that the contemporaneous impact of each of the federal initiatives is to increase multiple starts. Although not all of the estimated impacts are statistically significant, all are significant in an economic sense. The net contribution to multiple starts, in each year from 1971 to 1987, is shown in Summary Table 1. Also shown in Summary Table 1 are the starts initiated under each program, as reported by the CMHC.

- (1) The increase in the stock of multiple units attributable to each program as of year-end 1987, expressed as a ratio of the total starts initiated under the program, is as follows:

LD:	46.5 per cent
ARP:	18.7 per cent
CRSP:	40.9 per cent
MURB:	35.6 per cent

These figures, which follow directly from the data reported in Summary Table 1, indicate that 46.6 per cent of the starts initiated under the LD program represent a net increase in the

multiple housing stock as of year-end 1987. A like interpretation applies to the other figures.

- (2) In our model, temporary programs - such as the four federal initiatives - cannot permanently increase the stock of multiple units. As shown in Summary Table 1, the net impact after a program has expired is to depress multiple starts. This is due, in the main, to the fact that the increase in the stock of multiple units, a consequence of the program, serves to lower rents. Lower rents, in turn, serve to depress starts.
- (3) If the simulation exercises were extended to the indefinite future, the structure of the housing model implies that rents would continue to decline, thereby depressing starts. These induced effects would continue until the positive stimulus to the stock of multiple units was fully offset. The induced offset, through 1987, is small. This is due to the relatively modest decline in rents that accompanies the increase in the stock of multiple units. This result, in turn, can be traced to the existence of rent controls over most of the simulation period. Because of controls, the initial impact of an increase in the stock of multiples is to increase the vacancy rate.

- (4) Our estimates do not correct for the possibility that there is a shift from owner-occupied (condominium) to rental starts as a result of the federal initiatives. Data limitations preclude our investigating this possibility. If such shifting does occur, our estimates will understate the impact of the federal initiatives on rental (as distinct from multiple) starts.
2. Since a temporary federal initiative designed to stimulate rental starts does not alter the determinants of the stock demand for rental housing, these initiatives cannot permanently increase the stock of multiple units.
- (1) Nonetheless, by the end of 1987, the multiple housing stock is greater by 171,400 units as a result of the combined effects of the federal programs. This represents 4.4 per cent of the multiple housing stock that existed at year-end 1987.
- (2) The fact that the stock of multiple units at year-end 1987 is significantly greater as a result of the combined federal initiatives is due to the slow speed with which induced offsets occur. By year-end 1987, after the programs have all expired, rents are only 3.3 per cent lower than otherwise. As a result, multiple starts are depressed by only 3,000 units in 1987 relative to the level than would have occurred if none of the federal initiatives had been introduced.

(3) To identify a "high" estimate of the impact of the federal initiatives, we increase by one standard deviation the estimated coefficients of the capitalized subsidies variables in the multiple starts equation. To identify a "low" estimate, we reduce the estimated coefficients by one standard deviation. In the latter case, LD, ARP and CRSP exert no net impact on multiple starts, although MURB continues to do so.

(4) Under our "high" estimate of the impact of the federal initiatives on starts, the multiple housing stock is higher by 305,500 units at year-end 1987. Under our "low" estimate, the multiple housing stock is higher by 62,500 units. These figures represent 178 per cent and 36 per cent, respectively, of the addition to the stock implied by our preferred estimates.

3. The simulated impact of the combined programs on market-wide rents, vacancy rates, and the price and stock of single family homes for the period 1971 to 1987 are presented in Summary Table 2. These results reflect the interaction of our housing model with the FOCUS model of the Canadian economy. The key results are as follows:

(1) Rents fall gradually through 1983, and then decline more rapidly as a consequence of the abandonment of rent controls

in British Columbia. At year-end 1987, rents are 3.3 per cent lower than otherwise.

- (2) The vacancy rate rises steadily during most of the simulation period, peaking at 1.1 percentage points higher than otherwise in 1984-86.
 - (3) The price of single family homes is largely unaffected, rising slightly throughout the simulation interval. The stock of singles declines slightly in the 1980's, and is lower by 4,800 units at year-end 1987.
4. The impact of the combined programs on real output, employment and federal and provincial surpluses/deficits, for the period 1971 to 1987, is presented in Summary Table 3.
- (1) Real GDP rises during the period 1971 to 1984, and declines thereafter. The maximum increase, equal to \$695 million (1981 dollars) or 0.2 per cent of GDP, occurs in 1976. The slight decline in real GDP in 1985 and in subsequent years reflects the negative induced effects that act to depress multiple starts near the end of the time period. Some small declines also occur in business investment in non-residential construction during the 1980's. These are a consequence of a very slight increase in interest rates brought about by the increase in the stock of multiple units.

- (2) Employment rises in the 1970's and then declines in the 1980's. These effects are quite modest in size. The annual increase (or decrease) never exceeds 10,000 jobs. The impact on the unemployment rate is negligible.
- (3) The net impact of the federal initiatives is to reduce the federal and provincial deficits (or increase surpluses) from 1974 to 1979, always by a very modest amount. In effect, the induced expansion of GDP with the associated increase in tax revenues exceeds the direct costs of the initiatives. By 1983, the direct costs exceed any induced increase in tax revenues, and the net contribution of the federal rental initiatives is to increase the deficits at both levels of government. (Note that the direct costs of each of the programs take place over an extended period of time, and thus "survive" the formal expiry date for the programs.)
- (4) Residential construction expenditures (not shown) rise relative to the base case from 1971 to 1985, reflecting the increase in multiple starts. The maximum increase - equal to 3.1 per cent - occurs in 1981.
- (5) The impact of the combined programs is to raise the level of interest rates (not shown), but the amount is negligible.

5. Cyclical volatility -- as measured by the sample-period coefficient of variation for multiple starts and the sample-period standard deviation of de-trended real residential construction expenditures - decreases slightly as a result of the federal initiatives.

(1) The federal initiatives had no significant effects on the cyclical instability of the index of capacity utilization and the unemployment rate. This result is consistent with the very modest reduction in the volatility of multiple starts and residential construction noted above.

(2) The federal initiatives reduced the (negative) correlation between multiple housing starts and the nominal interest rate on mortgages during the period 1971 to 1987.

III. Auxiliary Findings

6. In assessing the impact of the federal initiatives, one must recognize and incorporate the fact that these programs were introduced or enriched at times when the market for rental units was weak. Our econometric estimates indicate that the stimulus provided by LD, ARP and CRSP is understated if no correction is made for the resulting simultaneous equation bias. Our findings are based on equations which have been purged of the simultaneous equation bias.

7. In the regression explaining multiple starts, the coefficients of the subsidies contained in LD, ARP and CRSP are constrained to have an identical impact. In other words, a subsidy with a present value of one dollar is constrained to have the same impact, regardless of whether the subsidy is delivered through LD, ARP or CRSP. The present values, of course, reflect all of the program features, including their constraints.
8. Clearly, rent controls play an important role in the market for rental housing during most of our sample period. We tested a variety of dummy variables designed to pick up the impact of rent controls in the multiple starts equation. In general, these attempts were unsuccessful, as documented in Appendix C. We include a very simple dummy variable that switches "on" in 1978:4, the time at which rent controls in Ontario were perceived as becoming permanent. Rent controls also influence multiple starts indirectly, since the expectation of the future level of rents affects the capitalized values of the subsidies associated with the federal programs. In addition, rent controls influence both the stock demand for multiple units (and hence vacancies), as well as the rate of change of rents in our econometric model.
9. Appendix A develops new approaches for evaluating (i) the benefits of the programs for investors and (ii) the costs of the programs for the government.

- (1) We derive an expression for the expected present discounted sum of the returns from a housing unit in terms of the specific subset of variables that define the terms and conditions of the different housing programs. This exercise provided a common framework for 'pricing' (both singly and in combination) a sequence of interest subsidies, loans and/or grants, tax allowances, and different types of rent restrictions. We also show in Appendix A that previous studies have, by ignoring principal payments and hence a changing equity balance, miss-specified the discount factors used in determining the present discounted value of a unit.
- (2) Previous estimates of housing program costs have generally determined the total cumulative decrease in government revenues and/or increases in government expenditures. This approach typically overestimates the net cost. Our approach is to view the government as an intermediary between investors and the capital market. According to this view, a subsidy to the investor (whether a direct interest subsidy, an interest free loan or a tax allowance that is recaptured on sale) is equivalent to a loan from the government to the investor at below market interest rates. The cost to the government is thus the difference between its borrowing rate and the rate on the implicit or explicit loan to the investor. The resulting annual cost under each program is described in Summary Table 4.

10. We were able to find no evidence of cross-substitutions between the stock demands for single and multiple units in the econometric investigation. Empirically, the stock demand for multiple housing units responds negatively to changes in (real) rents and positively to changes in real disposable income but shows no significant response to changes in the user cost of single-detached units. Similarly, the demand for single units is negatively related to the single-unit price and positively related to real disposable income but is insensitive to rents.
11. The model successfully tracks quarter-to-quarter movements in the price of single-detached units. Prices rise in response to an increase in real disposable incomes, and decline in response to an increase in either the stock of singles or the real interest rate. These price movements are thus governed primarily by fundamentals rather than, for example, by speculation.

Summary Table 1

Contribution to Multiple Starts of Individual Programs, 1971-1987

Year	LD		ARP		MURB		CRSP	
	Gross Starts ^a	Net Starts ^b	Gross Starts ^a	Net Starts ^b	Gross Starts ^a	Net Starts ^b	Gross Starts ^a	Net Starts ^b
1971	11.5	1.6	-	-	-	-	-	-
1972	8.8	2.3	-	-	-	-	-	-
1973	4.5	2.4	-	-	-	-	-	-
1974	2.5	5.0	-	-	2.1	1.9	-	-
1975	10.9	7.9	22.3	5.8	8.5	13.9	-	-
1976	-	2.1	25.1	8.4	35.2	17.4	-	-
1977	-	-0.2	57.1	5.6	82.3	16.0	-	-
1978	-	-0.3	18.2	3.7	80.1	16.4	-	-
1979	-	-0.3	-	0.7	76.5	10.5	-	-
1980	-	-0.3	-	-0.1	-	5.9	-	-
1981	-	-0.3	-	-0.1	61.5	36.9	-	-
1982	-	-0.3	-	-0.1	-	10.1	10.4	2.1
1983	-	-0.3	-	-0.1	-	-0.2	10.3	3.2
1984	-	-0.3	-	-0.1	-	-0.9	3.5	3.4
1985	-	-0.4	-	-0.2	-	-1.4	-	1.6
1986	-	-0.5	-	-0.3	-	-2.0	-	0.0
1987	-	-0.5	-	-0.3	-	-2.1	-	-0.1

Notes:

- (a) These are the starts (in thousands) initiated under each program, as reported in Table A of C.M.H.C., Schedule A - Terms of Reference, for "The Economic Impacts of Federal Rental Housing Initiatives".
- (b) These are the net starts, based on our estimated model of the housing market and simulations with the FOCUS model of the Canadian economy, reported in our Appendix D. (The depressing impact on starts after the expiry of the program is due, in the main, to the lower level of rents resulting from the prior increase in the multiple stock of housing.)

Summary Table 2

Impact on Housing Market of Combined Programs, 1971-1987

<u>Year</u>	<u>Rents</u>	<u>Vacancy Rate</u>	<u>Single Houses</u>	
			<u>Price</u>	<u>Stock</u>
1971	0.0	0.0	0.0	0.0
1972	0.0	0.0	0.0	0.0
1973	-0.1	0.0	0.0	0.0
1974	-0.1	0.0	0.0	0.0
1975	-0.3	0.1	0.1	0.0
1976	-0.3	0.2	0.2	0.0
1977	-0.3	0.4	0.2	0.0
1978	-0.3	0.5	0.1	0.0
1979	-0.4	0.6	0.1	0.0
1980	-0.4	0.8	0.0	-0.1
1981	-0.5	0.8	0.1	-0.5
1982	-0.5	0.9	0.2	-1.1
1983	-0.6	1.0	0.2	-1.8
1984	-0.9	1.1	0.1	-2.6
1985	-1.7	1.1	0.2	-3.5
1986	-2.5	1.1	0.2	-4.3
1987	-3.3	1.0	0.5	-4.8

Notes: Figures quoted for rents and prices refer to percentage changes. The vacancy rate is measures in percentage points, and the stock of singles is measured in thousands of units.

Source: Appendix D, Table 6.

Summary Table 3

Impact on Macroeconomy of Combined Programs, 1971-1987

<u>Year</u>	<u>Real GDP</u>	<u>Employment</u>	<u>Unemployment Rate</u>	<u>Government Surplus/Deficit</u>	
				<u>Federal</u>	<u>Provincial</u>
1971	20	0.3	0.0	-3	1
1972	47	0.6	0.0	-2	3
1973	66	0.9	0.0	0	5
1974	111	1.3	0.0	9	10
1975	415	4.8	0.0	51	37
1976	695	9.1	0.0	114	70
1977	667	9.2	0.0	132	76
1978	579	6.9	0.0	110	55
1979	397	2.5	0.0	60	18
1980	195	-2.0	0.0	-17	-27
1981	535	0.6	0.0	26	3
1982	572	3.4	0.0	-18	4
1983	251	0.1	0.0	-64	-63
1984	110	-3.9	0.0	-108	-134
1985	-12	-7.0	0.0	-182	-176
1986	-69	-8.1	0.0	-253	-185
1987	-12	-6.8	0.0	-233	-147

Notes: Real GDP and the government surplus/deficit are measured in 1981 dollars. Employment is measured in thousands, and the unemployment rate is measured in percentage points.

Source: Appendix D, Table 6.

Summary Table 4

Estimates of the Direct Program Costs of the Various
C.M.H.C. Rental Subsidy Programs

(NI&E Accounts (accrual) Basis, \$ Millions)

Year	Program							
	*Limited Dividend	**M.U.R.B.		+A.R.P.		++C.R.S.P.		TOTAL
	Federal	Fed.	Prov.	Fed.	Prov.	Fed.	Fed.	Prov.
1971	4.6	-	-	-	-	-	4.6	-
1972	6.5	-	-	-	-	-	6.5	-
1973	7.7	-	-	-	-	-	7.7	-
1974	8.9	-	-	-	-	-	8.9	-
1975	13.6	0.3	0.2	0.2	0.0	-	14.2	0.2
1976	13.8	1.9	1.0	2.6	0.5	-	18.3	1.5
1977	13.9	7.4	3.7	9.9	2.0	-	31.2	9.4
1978	14.0	16.5	8.2	17.7	3.5	-	48.2	11.7
1979	14.0	29.2	14.5	19.5	4.0	-	62.7	18.5
1980	14.0	39.9	20.4	25.0	5.0	-	78.9	25.4
1981	14.0	55.7	27.8	29.7	5.9	-	99.4	33.7
1982	14.0	69.4	34.7	32.3	6.4	1.2	117.4	41.1
1983	14.0	82.1	41.0	36.8	7.3	5.4	138.3	48.3
1984	14.0	93.7	46.9	46.4	9.3	15.1	169.2	56.2
1985	14.0	102.4	51.1	35.3	7.0	15.4	167.1	58.1
1986	14.0	101.3	50.8	47.9	9.6	23.2	186.4	60.4
1987	14.0	80.4	40.2	51.9	10.4	24.1	170.4	50.6

* Estimates of federal subsidies are derived from the Annual Non-Budgetary Funds authorized under LD, as recorded by Canadian Housing Statistics (CHS).

** The direct costs of M.U.R.B.'s are estimates of losses in personal income tax accruals to the federal and provincial governments associated with this program. The estimation technique is detailed in Appendix A.

+ Estimates of the direct costs of the Assisted Rental Program consist of federal subsidies to business and provincial subsidies to business resulting from "top ups" in Ontario and British Columbia (estimated at 20% of federal direct costs). The federal subsidies are estimated by the Annual Budgetary Expenditures under ARP (CHS).

++ Direct costs of CRSP are estimated by the Annual Budgetary Expenditures under CRSP (CHS).

May 16, 1989

Appendix A

**CALCULATING THE CAPITALIZED VALUES
FOR INVESTORS AND THE COSTS FOR GOVERNMENT
OF THE FEDERAL RENTAL INITIATIVES**

by

**George Fallis
Arthur Hosios
Gregory Jump
James E. Pesando
Lawrence B. Smith**

Institute for Policy Analysis

University of Toronto

Calculating the Capitalized Values for Investors and the Costs for Government of the Federal Rental Initiatives

The goal of this subproject is two-fold: (1) to construct time series representations of the values for investors of the same housing unit under different federal housing programs as well as in the absence of such programs; and (2) to estimate the corresponding annual costs for government of each of these housing programs. The first series will be used as explanatory variables in our housing starts equation for multiple units; the second series will be used in our macro simulations as inputs to the government's budget.

1. Introduction

If data on the stock prices of housing units were available corresponding to each of the tax regimes and housing programs of interest, our first task would be finished. There would be no need to calculate the present discounted value of the associated streams of rents and subsidies. Under the assumption of an efficient housing market, the observed values would already fully capitalize the different features of the units and the housing programs under consideration. Unfortunately, the available price data are incomplete.

Table 1 lists the average unit value for projects approved under three different rental programs. Data for projects covered by the Multiple Unit Residential Building program (MURB) are unavailable. The second column of Table 1 lists the average first mortgage per approved unit under the Limited Dividend Program (LDP) 1968-75, the Assisted Rental Program (ARP) 1975-78, and the Canada Rental Supply Program (CRSP)

1982-85. Given these first mortgage numbers, the estimated project values were then determined by using the historical average loan-to-value ratios for LD, ARP and CRSP projects, specifically, 95%, 90% and 56%, respectively.

It is easy to see which numbers are missing. We would clearly like to know the market values of units not covered by these programs, with and without capital cost allowances and/or tax offsets, throughout the period 1968 to 1988. Moreover, we may also wish to make hypothetical comparisons. For example, what would the unit values have been under LD, ARP, MURB or CRSP in 1980 as compared to the value of the same project without any program?

The solution to these problems is to construct measures that should be highly correlated with the unobservable price series of interest. In this project, we calculate the expected present discounted value of the stream of returns associated with a housing unit under different programs and at different points in time. The material in Sections 2-6 below summarizes our approach and data, and describes the results.

On the cost side, the available program data are also incomplete. Canadian Housing Statistics records expenditures and disbursements by CMHC under past and present programs. The data from 1970 onward for the programs of interest are described in Table 2. Since the cost to the government of a loan to an investor is the difference between the government's lending and borrowing costs, rather than the loan itself, it follows that Budgetary Expenditures, rather than Non-Budgetary Funds, is the more appropriate measure of program cost. However, Budgetary Expenditures information is available for only two programs, ARP and

CRSP. As a result, we will have to develop our own estimates of the program costs in the case of LD and MURB. These estimates are presented in Section 7.

1. The Conceptual Framework

In an efficient equity market, the price of a stock is equal to the expected present discounted value of the stream of (anticipated) dividends associated with that stock. Similarly, in an efficient housing market, the price of a unit of housing stock is equal to the expected present discounted value of the stream of (anticipated) rents associated with that unit.

To start, let R_t denote the rental price of a unit at time t and let r denote the tax rate. In equilibrium, the marginal benefit from holding a unit of housing, $(1-r)R_t$, must equal the marginal cost. (If this is not the case, the price will be bid to the point where this equality holds.)

To describe the marginal cost of a housing unit, let V_t denote the stock price of the unit at t ; let δ denote the depreciation rate; let i_t denote the nominal borrowing rate; let η_t denote the nominal expected rate of return on equity; and let μ_t denote the operating cost associated with a unit (as a percentage). Therefore, in equilibrium,

$$(1-r)R_t = (\delta + (1-r)[i_t \ell_t + \eta_t e_t + \mu_t])V_t - (V_{t+1} - V_t), \quad (1)$$

where $\ell_t = L_t/V_t$ is the loan-to-value ratio and $e_t = E_t/V_t$ is the equity-to-value ratio. The marginal cost of owning a unit on the right-hand-side of (1) includes depreciation, operating costs and an appropriately

weighted measure of opportunity costs, net of any expected asset appreciation. Capital gains taxation, capital cost allowances, soft cost deductions and different tax rates on debt and equity are ignored to simplify the exposition.

This asset-pricing equation gives

$$V_t = \frac{(1-r)R_t + V_{t+1}}{1+\delta+(1-r)[i_t l_t + \eta_t e_t + \mu_t]} \quad (2)$$

In turn, updating this equation gives an expression for V_{t+1} as a function of R_{t+1} and V_{t+2} . Then, recursively substituting for V_{t+1} , V_{t+2} , etc., in (2), we get

$$V_t = \sum_{j=0}^{\infty} (1-r)R_{t+j} / (1+d_{t+j}) \quad (3)$$

that is, the stock price of a unit of housing equals the expected presented discounted value of the (after-tax) stream of rents, where the discount factors are defined by

$$(1+d_{t+j}) / (1+d_{t+j-1}) = 1+\delta+(1-r)[i_{t+j} l_{t+j} + \eta_{t+j} e_{t+j} + \mu_{t+j}] \quad .$$

Observe that (3) can also be written as

$$V_t = \sum_{j=0}^{T-1} (1-r)R_{t+j} / (1+d_{t+j}) + V_{t+T} / (1+d_{t+T}) \quad .$$

Thus, the value of a unit today equals the discounted stream of its after-tax returns independent of whether that unit is held for T periods and sold at the anticipated price V_{t+T} , or held for an infinite number of periods.

3. An Alternative Formulation

For any given sequence of expected rents and discount factors, R_{t+j} and d_{t+j} , equation (3) can be used to estimate the initial project value, V_t . Nevertheless, the equations we employ are slightly different. In particular, we have chosen to reformulate the problem in a manner that highlights a more conventional notion of after-tax cash flows as the relevant return per period to be discounted. This is because the different housing programs under consideration are expressed, either explicitly or implicitly, in terms of these conventional after-tax cash flows.

To proceed, let $OC_t = \mu_t V_t$ denote the operating costs of a project in period t , let $I_t = i_t l_t V_t$ denote the interest payments in t and let P_t denote the principal payment in t . Thus, in the absence of capital cost allowances, offsets and subsidies, income after operating costs and interest is given by $R_t - OC_t - I_t$, and so the after tax cash flow is

$$ATCF_t = (1-r)(R_t - OC_t - I_t) - P_t .$$

Then, rearranging the terms in equation (1), we have

$$(1-r)R_t = (\delta + (1-r)\eta_t e_t)V_t + (1-r)[OC_t + I_t] - (V_{t+1} - V_t),$$

$$(1-r)(R_t - OC_t - I_t) = (\delta + (1-r)\eta_t e_t)V_t - (V_{t+1} - V_t) ,$$

$$(1-r)(R_t - OC_t - I_t) - P_t = (\delta + (1-r)\eta_t e_t - p_t)V_t - (V_{t+1} - V_t) , \quad (1')$$

where $p_t = P_t/V_t$ is the principal-to-value ratio in t . Note the P_t appears on the left-hand-side of (1') as it reduces the after-tax cash flows of equity holders; P_t appears on the right-hand-side because equity

holders have, at par, increased the value of their claim by paying down the principal of the loan.

Solving (1') we then get

$$V_t = \sum_{j=0}^{\infty} [(1-\tau)(R_{t+j} - OC_{t+j} - I_{t+j}) - P_{t+j}] / (1+d_{t+j}) , \quad (3')$$

where

$$(1+d_{t+j}) / (1+d_{t+j-1}) = 1 + \delta + (1-\tau)\eta_{t+j}e_{t+j} - P_{t+j} . \quad (4)$$

Thus, while equations (3) and (3') yield identical values for V_t , equation (3') is expressed in terms of the same set of variables that is also used to define the different government housing programs. For this reason, (3') is the starting point for the calculations described below.

4. Calculating Present Discounted Values

Using (3'), the value at time t of a building held for T periods is

$$V_t = \sum_{j=0}^{T-1} ATCF_{t+j} / (1+d_{t+j}) + V_{t+T} / (1+d_{t+T}) , \quad (5)$$

where V_{t+T} is the anticipated sales price of the unit at time $t+T$ and the discount factors are again described by (4). Attention will be focused on 10 year holding periods. A different version of the right-hand-side of (5) will be calculated for each quarter of the sample period, from 1970 to 1988, corresponding to the tax regime and housing program in effect during that quarter.

The tax regimes under consideration have four dimensions:

- (i) a capital cost allowance, with or without;
- (ii) tax offsets (when the owner can deduct losses on the project against other income);

- (iii) a recapture tax (when the owner must pay a tax, at the time of sale, on the difference between the initial cost of the structure and the undepreciated balance at that time);
- (iv) a soft cost write-off (when the owner can deduct all of his soft costs as expenses during the first year - otherwise, the owner depreciates his soft costs over time in exactly the same manner as he depreciates the building).

The MURB program, in particular, will be modeled as a capital cost allowance with both tax offsets and soft costs write-offs.

The housing programs include:

- (i) the Limited Dividend Program (below market first mortgages);
- (ii) the Assisted Rental Program (annual grants and/or loans per unit) and;
- (iii) the Canada Rental Supply Plan (an interest free loan per unit in the first year only).

There are 3 different versions of Assisted Rental Program, ARP 1975, ARP 1976-77 and ARP 1978, and two separate provincial programs (top ups) in the case of ARP 1976-77.

The actual formulae used to represent these programs are succinctly described in Appendix A1. To describe how these calculations are performed, we refer directly to (5) above, which is basically the 'no program' case.

To determine the expected present value of the after-tax cash flows for a 10 year holding period plus the discounted sales value in the tenth year for a project that starts in the 1st quarter of 1970, denoted 1970(1), we need to forecast the annual rents, operating costs, interest

and principal payments (given the current mortgage interest rate and a given amortization period) for 1971(1), 1972(2), ..., 1980(1), in addition to the expected selling value at 1980(1). With these numbers in hand, the computational exercise is straightforward.

To generate the entire time series corresponding to 'no program', the same exercise must be performed for each quarter, from 1970(1) to 1988(3). For example, to determine the expected present value of the after-tax cash flows for a project that starts in the 3rd quarter of 1979, we would again need to forecast the annual rents, operating costs, interest and principal payments for 1980(3), 1981(3), ..., 1989(3), and the expected selling value at 1989(3).

It is important to recognize that the 10 year forecasts made at times t and $t+k$ will, in general, differ. That is, to determine the PDV of a project that begins in 1975(2) we need to forecast the annual rents from 1976(2) to 1985(2). To determine the PDV of a project that begins in 1978(2), we need to forecast the annual rents from 1979(2) to 1988(2). Nevertheless, the forecasted rents for the overlapping period, 1979(2) to 1985(2), from the perspective of 1975(2) and from the perspective of 1978(2) should differ because the information available to investors at times 1975(2) and 1978(2) differ. The next section describes our forecasting techniques.

5. Data and Forecasts

To calculate the present discounted value of the after-tax cash flows for each of the possible tax regime/housing program combinations, we need to, first, specify the initial conditions defining a new unit in

each quarter of the sample period and, second, corresponding to each such new unit, forecast the relevant project variables over its holding period.

Defining a Project

The data in Table 1 (appropriately interpolated and extrapolated) are used to set the value of a representative unit in each period. See Figure 1a. Admittedly, this data series does not describe the actual values of a given housing unit over time. We continue to employ it for two reasons. First, it will likely be highly correlated with the unobservable series of interest. And second, as our primary goal is to identify the relative values of a unit under different tax/housing programs, it is not essential that we correctly describe the absolute values.

Taking the unit value in period t from the series depicted in Figure 1a, this value is decomposed, respectively, into land, structure and soft costs in the following proportions: .25, .65 and .1 in 1970(1); .33, .57 and .1 in 1988(3); and linearly interpolated proportions in between. The changing proportions for land and structure costs reflect the relative increases in land costs over the sample period.

The loan to value ratio is .9, with two exceptions. Under the LDP it is taken to be .95; under CRSP, it is taken to be .6. The amortization period of the loan is 35 years. The first year rent and operating costs are fixed proportions, .14 and .6, of the given unit value. The economic rate of depreciation (for discounting purposes) is 2% per annum. The tax rate is 50%.

The various fixed proportions specified above are based on the 'typical' or 'average' project described in the literature and by investors. By pooling information from different sources in this way to define a representative unit, however, it is entirely possible that the resulting unit will be somewhat different from any actual unit. In fact, we discovered that the requisite 5% to 10% returns on equity quoted in the housing program descriptions, even in real terms, are much lower than those needed to make these programs attractive to investors contemplating this representative unit. Therefore, to calibrate the model, the nominal rate of return on equity, where required to calculate program subsidies, was adjusted so that the correct rank-ordering of the programs would result; i.e., a common rate of return was chosen, 25%, so that the present value of the after-tax cash flows from a unit under any program exceeds its present value in the absence of a program. While this nominal rate may appear high, it should be noted that, in real terms, it is much closer to the rates experienced by developers during the sample period; during that period, 5-10% nominal rates of return translate into negative real rates.

Forecasting

During each quarter of the sample period, our hypothetical investor will have to forecast (i) rents, operating costs and nominal interest rates over a 10 year holding period and (ii) building and land values at the sales date.

The estimated forecasting equations for interest rates and unit values are as follows (standard errors are in brackets):

Mortgage Interest Rates

$$i_t = 1.0195 + 0.91738i_{t-1} \quad R^2 = .845$$

$$(0.57121) \quad (0.4622E-01)$$

Project Unit Values

$$\ln(V_t) - \ln(V_{t-1}) = 0.010698 + 0.59262(\ln(V_{t-1}) - \ln(V_{t-2})) \quad R^2 = .35$$

$$(0.004186) \quad (0.10736)$$

The latter equation was estimated by a maximum likelihood iterative technique correcting for first-order serial correlation (the final value of ρ was .19).

The estimated parameters in both equations are statistically significant. The R^2 value for the unit value equation is low but not surprising as this general form of regression equation (a difference in natural logs on a difference in natural logs) typically gives low R^2 values. What is important for our analysis is that these equations yield stable, unbiased forecasts. See Figure 1a. The alternative naive approach to expectations formation, i.e. constant interest rates and a constant rate of growth of unit value, generates unacceptable forecast errors.

To illustrate how these equations are used, consider a linear forecasting equation that describes some variable z as a function of the previous value of z ; specifically, $z_t = \alpha + \beta z_{t-1}$. We would employ this equation as follows. Suppose the actual value of z at $t=0$ is x . Then, given the information available at $t=0$, the estimated values of z at $t=1,2,3$ are $z_1 = \alpha + \beta x$, $z_2 = \alpha + \beta(\alpha + \beta x)$ and $z_3 = \alpha + \beta(\alpha + \beta(\alpha + \beta x))$,

respectively. Now, suppose the actual value of z at $t=1$ turns out to be y (in general, of course, y will not exactly equal $\alpha+\beta x$). Then, given the information available at $t=1$, the estimated operating costs at $t=2,3,4$ are $z_2 = \alpha+\beta y$, $z_3 = \alpha+\beta(\alpha+\beta y)$ and $z_4 = \alpha+\beta(\alpha+\beta(\alpha+\beta y))$, respectively. For the forecasts depicted in Figure 1a, which are from the perspective of 1970(1), the starting unit values are the observed values this quarter and last, 1970(1) and 1969(4).

Expected future rents are .14 of forecasted unit values. The expected rate of growth of operating costs is 25% higher than the forecasted rate of growth of rents. (This reflects the higher rate of growth of operating costs experienced throughout the sample period.) Expected future building and land values are fixed proportions of the expected future unit values. As described above, these proportions adjust over time, falling from .65 to .57 for buildings and rising from .25 to .33 for land. The latter adjustments induce a higher rate of growth for land relative to structure values.

Some further comments on these forecasts are in order: First, when forecasting future rents, operating costs, etc., our investors do not formally solve a structural model of the housing market for these future values. Rather, they use the forecasting equations for mortgage interest rates and project unit values described above. These forecasting equations were based on data from the entire sample period to maximize their informational content for forward-looking investors. The unit value equation, in particular, can be viewed as a reduced-form equation of the true model of the housing market, subject to the caveat that the remaining forcing variables (national income, the housing stock) are

relatively slow moving. This is as close as this part of our model comes to implementing the rational expectations assumption.

Second, additional forecasting equations could have been estimated for rents and operating costs. However, after some experimentation, we concluded that adding forecasting equations to the model only increased the variation in the present value series that could not easily be accounted for without increasing the explanatory power of these series in the housing starts equations. It is much simpler (and much closer to the way actual investors operate) to forecast rents as a fixed fraction of forecasted unit values, and to assume that operating costs grow 25% faster.

6. Modeling the Programs

Our base case corresponds to the description of taxes (including recapture), offset and soft cost provisions, and permissible capital cost allowances that is summarized by (A) and (B) below and in Figure 2.

(A) Taxes

1970(1) to 1971(4) Capital cost allowance with offset against other income

Recapture on sale taxed at 25% (i.e. 50% of the tax when buildings cannot be rolled over)

1972(1) to 1987(4) Capital cost allowance without offset
Recapture on sale taxed at 50%

(B) Depreciation and Soft Costs

1970(1) to 1978(4) Allowable depreciation is 7.5% (i.e. half way between 5% and 10%)
Soft costs write-off in first year

1979(1) to 1987(4) Allowable depreciation is 5%
Soft costs depreciated with building

Prior to 1972(1), investors could postpone paying the recapture tax when selling a building. This was done by purchasing a second building and transferring (rolling over) the depreciated portion of the first building to the second building. However, postponing a tax does not mean that it is never paid. By reducing the recapture tax from 50% to 25% between 1970(1) and 1971(4) we are taking account of the fact that the present discounted value of the (postponed) tax is still positive but less than what it would have been had roll-overs been impossible.

The present value series corresponding to the base case, from 1970(1) to 1987(3), is listed in column 1 of Table 3 and is depicted in Figures 3-7. Each point in this series shares two features. First, the anticipated annual after-tax cash flows typically start out negative but become larger (and eventually positive) over the 10 year holding period of the building. The major exception occurs when soft cost write-offs are possible, so that the first year after-tax cash flow may exceed the second year value. Second, of the two components of a project's present value, i.e., the discounted sum of after-tax cash flows and the discounted after-tax sales proceeds, the former term generally constitutes less than 25% of the sum.

Notice, in each quarter throughout the sample period, an investor takes a 35-year mortgage at the current rate and is committed to that rate for 10 years. However, when interest rates rise quickly, developers/buyers are likely to shorten the terms to maturity of their mortgage loans. We do not allow for this type of endogenous response.

In consequence, the dramatic fall throughout 1980(2) to 1981(3) in our base case series, in response to the corresponding rapid rise in mortgage interest rates (see Figure 1b), is likely greater than the decrease that actually occurred during that period.

During the 1980's, the discounted after-tax cash flows are negative. As a result, the effects of increases in interest rates are exacerbated because of the absence of tax offsets. This is clearly illustrated by the difference in present values between the base case and the MURB series in 1981(3) and 1981(4).

On the other hand, the substantial increase in the base case series between 1982(1) and 1982(4) is due only in part to the decrease in interest rates. Rather, it is mostly explained by increases in the expected after tax sales proceeds which, in turn, are due to increases in the forecasted rate of land and building appreciation as the unit value series passes the kink in 1982(1) (see Figure 1a).

Multiple Unit Residential Building Program

The MURB program corresponds to a combination of the base case, (A)-(B), and (C) below, and hence (A)-(C) in Figure 2. The present value series is presented in the blocked out area of column 2 of Table 3 and is depicted in Figure 3 relative to the base case. As expected, introducing the MURB program makes investment in multiple unit project more attractive than otherwise.

(C) MURB

1974(4) to 1979(4) Capital cost allowance with offset
Soft costs write-off in first year

1980(4) to 1981(4) " .

Limited Dividend Program

The LD program corresponds to a combination of the base case, (A)-(B), and (D) below, and hence to (A)-(B) and (D) in Figure 2. The present value series is presented in the blocked out area of column 3 of Table 3 and is depicted in Figure 4 relative to the base case.

(D) LD

1970(1) to 1975(4) Mortgage interest rate 2% below market; first year rent set to yield prespecified return on equity; subsequent rent increases to cover operating cost increases

Assisted Rental Program

The ARP program corresponds to a combination of (A)-(B) and (E) below, and hence to (A)-(B) and (E) in Figure 2. The present value series for ARP by itself, ARP in Ontario and ARP in B.C. are presented in the blocked out areas of columns 3, 4 and 5 of Table 3, respectively, and are depicted in Figure 3 relative to the base case.

(E) ARP

1975(2) to 1976(1) ARP 1975 - non-repayable, non-taxable annual grants per unit; subject to a ceiling, first-year grant subsidizes rent to yield prespecified return on equity; subsequent grants decrease by fixed amounts; rents cannot increase by more than the change in operating costs plus the decrease in the ARP grant

- 1976(2) to 1978(1) ARP 1976 - repayable, interest free loan per unit per annum; subject to a ceiling, the first-year loan subsidizes rent to yield a prespecified return on equity; subject to a floor, subsequent loans decrease to yield the same return each year; loan repaid at the end of 10 years'
- Ont. - up to \$600 grant after ARP ceiling
B.C. - up to \$600 grant before ARP and \$1200 loan after ARP ceiling
- 1978(2) to 1978(4) ARP 1978 - repayable, interest-bearing loan per unit per annum; subject to a project-specific ceiling, the first-year loan subsidizes rent to yield a prespecified return on equity; subsequent loans decrease by 5% of the difference between previous year's interest plus principal payments and the previous years loan; loan repaid with accumulated interest at the end of 10 years

Note: (i) The depreciation rate applied to ARP projects for capital cost allowance purposes is 8.75% to reflect the mix of building structures specific to that program.

- (ii) Outside of Ont. and B.C., the ceiling for ARP 1976 is \$1200 in 1976(2)-1977(1) and is \$900 in 1977(2)-1978(1). The corresponding ceilings in Ontario and B.C. are \$1200 and \$1050 to account for the relative impacts of Toronto and Vancouver where the \$1200 ceiling was maintained.

Canada Rental Supply Program

The CRSP program corresponds to a combination (A)-(B), and (F) below, and hence to (A)-(B) and (F) in Figure 2. The present value series is presented in the blocked out area of column 7 of Table 3 and is depicted in Figure 6 relative to the base case.

(F) CRSP

- 1982(1) to 1985(1) CRSP - a repayable, one-time interest free loan that, subject to a ceiling, equals the difference between 80% of the unit cost and the first mortgage - participation of Ontario and B.C. (effectively throughout the period) changes the ceiling from \$7500 to \$10500

Toronto-Hamilton plus Vancouver-Victoria account for about 40% of CRSP

units, with a \$15000 per unit ceiling, while the \$7500 ceiling is employed in the rest of Canada.

The ARP program combined with the MURB program corresponds to (A)-(C) and (E) above. The present value series for ARP with MURB by itself, ARP with MURB in Ontario and ARP with MURB in B.C. are presented in the blocked out area of column 8, 9 and 10 of the Table 3, respectively, and are depicted in Figure 7. Comparing Figures 5 and 7, it is clear that MURB made the ARP program even more attractive.

7. Estimating the Cost of Each Rental Program

The precise cost to the federal government of each of the subsidy programs under consideration is not generally known. We shall therefore describe several different ways to estimate these costs. Our first method for developing a measure of program cost is applied to all of the programs and uses our present value series. Three steps are involved:

- (1) Take the difference between the present value of a project with and without a particular subsidy program, ΔPV , as the present value of repayable interest free loan from the government to the investor that the government finances by issuing a perpetuity. In this case, $\Delta PV = X/i$, where X is the government's cost per period and i is the rate at which the government borrows.
- (2) Multiply this number times the number of starts under the program in question, S , to get the total present value of the cost of this program, $(\Delta PV)S = SX/i$.
- (3) Convert these present value numbers into costs per period by multiplying by the rate of interest, i . We will use a quarterly

rate of 2.5%. These quarterly costs are born by the government during each year that a unit is being subsidized (45 years in the case of LD, 10 years in the case of ARP and CRSP and, assuming a 10 year holding period, 10 years in the case of MURB as well).

To illustrate, consider a program that lasts for 3 quarters and that is summarized as follows:

Quarter	Starts	ΔPV	Cost/Quarter
1	10	4	1.0
2	15	5	1.875
3	20	6	3.0

Even though the program itself lasts for only 3 quarters, the commitment to units started under the program is typically much longer. (For example, while ARP ran from 1975 to 1978, units covered by ARP generally received subsidies for 10 years; hence units built under ARP 1975 imposed costs on the government during 1975-1985 while those built under ARP 1978 imposed costs during 1978-1988.) Thus, suppose each unit in our example is subsidized for 4 quarters. In this case, the total cost will be the sum of 1 in periods 1 to 4 and 1.875 in periods 2 to 5 and 3.0 in periods 3 to 6, so that

Quarter	Total Cost/Quarter
1	1.0
2	2.875 (= 1. + 1.875)
3	5.875 (= 2.875 + 3.)
4	5.875
5	4.875 (= 5.875 - 1.)
6	3.0 (= 4.875 - 1.875)
7	0.0 (= 3. - 3.)

In the tables described later below, the annual program costs determined using the method above are list under the column heading PV. The following two features of the underlying present value series are

relevant: first, these series are determined from an investor's perspective and hence employ a variable risk-adjusted rate of return that is higher than the government's cost of funds (taken here to be 2.5% per quarter, on average); and second, in the case of LD, ARP and CRSP, they reflect the fact that these programs impose lower rents and/or rent increases than would otherwise occur. Thus, to the extent that the difference in these present value series is a relevant government cost measure, the resulting numbers should be viewed as providing a lower bound; i.e., higher discount rates and lower rents translate into lower present value series which, in turn, underestimate the cost to government.

Depending on the available data, one or more alternative methods for estimating costs will be described below. The main alternative to using the present value series is to view the government as acting as an intermediary between the capital markets and investors; the government borrows at the market rate and either lends at a lower rate to investors (LD, ARP, CRSP) or uses these funds to offset tax revenue losses (MURB). Interestingly, the numbers generated by these different approaches to estimating subsidy costs are not very different.

MURB

As an alternative to using our present value series, consider the following approach to estimating the cost of MURB.

Suppose a project structure costs \$100. With a 7.5% capital cost allowance and a 50% tax rate, the loss in tax revenue will be \$3.75 in the first year, \$3.47 ($-3.75 \times .925$) in the second year, \$3.21 ($-3.47 \times .925$)

in the third year, etc.. Suppose this building is sold at the end of ten years, at which point the recapture tax returns to the government the sum of its tax losses over the ten years. Therefore, from the government's point of view, this MURB certificate entitles the investor to an interest free loan of \$3.75 for ten years, of \$3.47 for nine years, of \$3.21 for eight years, etc.. Assuming a 10% rate of interest, the real cost to the government is the foregone interest payments, that is, \$0.375 in the first year, \$0.722 ($-.375 + .347$) in the second year, \$1.043 ($-.722 + .421$) in the third year, etc..

Applying a capital cost allowance of 7.5%, a tax rate of 50% and a further discount factor of .61 (to convert project costs to structure costs - .61 is the average fraction (1975-1981) used earlier in developing our present value series) to the unit costs in 1975-1981 (from Table 1 and Figure 1a), the calculation described above is performed for each of the multiple starts under the MURB program in that period.

The basic starts and unit cost data are described in Table 4. To determine the interest costs from 1975 to 1984 due to the first year of MURB-1975, we proceed as follows: .61 times unit costs, 23140, gives structure cost 14115; .075 times 14115 gives the CCA, 1058.7; .5 times 1058.7 gives the tax revenue loss, 529.3; .1 times 529.3 gives the interest cost per year of borrowing 529.3 for 10 years to offset the current tax revenue loss; and 52.93 times the number of starts, 8517, gives the total interest cost per year, 451 in 1000's.

To determine the interest costs from 1976 to 1984 due to the second year of MURB-1975, multiply 451 by .925, which gives 417 as the annual interest for nine years. To determine the interest costs from 1977 to

1984 due to the third year of MURB-1975, multiply 417 by .925, which gives 386 as the annual interest for eight years. The remaining interest costs, for each year of MURB and a 10 year holding period, is calculated in a similar manner.

To determine total costs, just add-up the annual interest costs: 451 in 1975; $451 + 417 + 2067 = 2935$ in 1976; $451 + 417 + 386 + 2067 + 1912 + 5833 = 11066$ in 1977; and proceed similarly to the end of 1984. For 1985, the cost equals $140654 + 1025 + 3126 + 3566 + 3979 + 4358$ minus 3194 (= the cumulative interest payments due to MURB-1975), which gives 153514. For 1986, the cost equals $153514 + 2892 + 3298 + 3680 + 4031$ minus 14919 (= the cumulative annual interest payments due to MURB-1976), which gives 152496. The 1987-88 costs are determined in the same way.

The results are described in Table 5. The PV column lists the annual costs of MURB based on the difference in the associated present value series, while the Alt column lists the costs using the alternative method described above. It is clear that either approach to estimating the opportunity cost of a program will dominate the standard practice of simply totaling the government's outlay. Given the difference in methods, however, these numbers are remarkably similar. The numbers in the Alt column will be used in later simulations.

LD

As an alternative to using our present value series, consider the following approach to estimating the cost of LD.

The Limited Dividend program is basically a 2% annual subsidy on the first mortgage of a project. Thus, add 2% of the first mortgage in year t (from Table 1 for LD) times the corresponding number of starts per year under LD to the cumulative program costs in years t to $t+44$ (assuming a 45 year mortgage). The annual estimated costs using the present value series and this alternative approach are listed in the PV and Alt columns in Table 6. Note that the simulations require cost estimates for 1971 to 1987 (see Appendix D).

There are three reasons for the difference between the PV and Alt columns. First, the discount rate employed by investors is variable and generally greater than that employed by the government. Second, investors are subsidizing renters through program mandated rent restrictions, which lowers their PV series with LD relative to the no-LD series. Third, costs are estimated very differently in PV and Alt columns; we generally favour the alternative method over the difference in PV series, but are nonetheless interested in comparing the relative magnitudes of these numbers.

A third estimate can be made using the Non-Budgetary Funds for LD from Canadian Housing Statistics that are listed in Table 2. Applying the 2% subsidy to the LD loan figures in Table 2, gives (in millions) 4.64 ($-.02 \times 231.9$) from 1971 to 2025, 1.89 ($-.02 \times 94.7$) from 1972 to 2026, 1.19 ($-.02 \times 59.5$) from 1973 to 2027, etc.. The resulting total annual

costs are listed in Table 5 under the heading CHS. These costs are very close to the mean of the PV and Alt numbers. The numbers in the CHS column will be used in later simulations.

ARP

Corresponding to the cost calculation using the present value series, column PV in Table 7 includes the Ont. and B.C. top-ups while column PVF represents the Federal ARP contributions alone. As an alternative, consider the following approach to estimating the cost of ARP.

For 1975-78, take the maximum first-year ARP grant/loan to be \$900., \$1200., \$1050. and \$820, respectively. (The last number equals .027 times the average first mortgage in 1978, which is \$30230 from Table 1.) Suppose these grant/loans are reduced by equal amounts over 10 years. The 1975 program involves non-repayable grants, which is equivalent to the government paying interest on a perpetuity corresponding to each of the 10 sequential grants per project. The 1976-77 loans are non-interest bearing but repayable after 10 years, which basically has the equivalent cost (foregone interest) implications to MURB. The 1978 loans are repayable, interest bearing loans, and so the repayment must be taken into account. Assuming that the typical grant/loan is 80% of the maximum, the resulting annual cost figures are listed in column Alt in Table 7.

Table 7 also lists the Budgetary Expenditures for ARP from Table 2 in column CHS. The numbers in columns Alt and CHS are not too different, except in 1987. This may in part be explained by the fact that the Non-

Budgetary Funds noted for ARP in 1981-1987, and listed in Table 2, represent supplementary assistance and deferrals of interest payments made to assist clients suffering adverse economic conditions. These payments and deferrals were clearly unanticipated; they are reflected in the corresponding ARP Budgetary Expenditures in column CHS but have been ignored in our determination of costs in columns PV, PVF and Alt. The numbers in column CHS will be used in later simulations.

CRSP

As an alternative to the difference in present values under this program, which is recorded in column PV in Table 8, we also estimated the cost/year in years t to $t+9$ of a CRSP loan in year t as the forgone interest on that loan. Specifically, this interest cost was estimated as 10% of the average Federal CRSP loan in Ontario times the number of CRSP starts in year t . The resulting total annual cost is described in column Alt.

The rationale for using the Ontario number is simple. The CORSP data indicates the average Federal Ontario CRSP loan (\$5485). Since this number is also representative of B.C., since Ontario and B.C. together dominate the sample, and since the remaining sample figures we have for projects elsewhere in Canada are in the same range, we decided to go with the Ontario figure.

Table 8 also lists the Budgetary Expenditure data for CRSP from Table 2 in column CHS. These data indicate that our PV and Alt series perform best in the early years, 1982-1985. The continuous rise in the CHS numbers after the end of CRSP, in 1985, suggests that additional

expenditures were incurred that are not reflected in our CRSP starts and program data. The numbers in column CHS will be used in later simulations.

Appendix A1

Formulae for Determining the Expected Present Discounted Value of Different Tax and Housing Programs

- I. The following variables are known at the beginning of quarter t ; they are either pre-specified (possibly dependent on the program under consideration) or derived from the data.

t = purchase date of project

$t+T$ = sale date

$V(t)$ = cost of a new project (unit) at t

$L(t)$ = land value at t

$B(t)$ = building value at t

$EQ(t)$ = equity (as fraction)

$MR(t)$ = mortgage rate (assume 20 yr. amortization)

$(1-EQ(t))V(t)$ = amount borrowed

$R(t+1)$ = first year rent

$OC(t+1)$ = first year operating costs

r = income tax rate

\hat{r} = capital gains tax rate

δ = depreciation rate of building

σ = selling costs (fraction of value)

ϵ = allowable yield on equity for rental programs

- II. The following variables need to be calculated from the perspective of period t .

$L^e(t+T)$ = expected land value at $t+T$

$B^e(t+T)$ = expected structure value at $t+T$

$I(t+i)$ = interest payment at $t+i$, $i=1, \dots, T$

$P(t+i)$ = principal paid at $t+i$ "

$R(t+i)$ = rent paid at $t+i$ $i=2, \dots, T$

$OC(t+i)$ = operating costs at $t+i$ "
 $DEP(t+i)$ = building depreciation at $t+i$ $i=1, \dots, T$
 $NDEP(t+T)$ = non-depreciated bulding at sale
 $d(t+i)$ = discount rate applied in period $t+i$

III. Some Identities and Definitions

$V(t) = L(t) + B(t)$
 $DEP(t) = \delta B(t)$
 $CCA(t+i) = \text{capital cost allowance at } t+i$
 $NDEP(t+1) = B(t) - CCA(t+1)$
 $DEP(t+i) = \delta NDEP(t+i-1) \quad i=2, \dots, T$
 $NDEP(t+i) = NDEP(t+i-1) - CCA(t+i) \quad i=2, \dots, T$

IV. Four Basic Present Value Calculations

$Z1(t+i) = \text{net revenue after interest at } t+i$
 $\quad = R(t+i) - OC(t+i) - I(t+i)$
 $Z2(t+i) = \text{taxable income in } t+i$
 $ATCF(t+i) = \text{after tax cash flow in } t+i$
 $\quad = Z1(t+i) - P(t+i) - r(Z2(t+i))$

No capital cost allowance and no tax offset:

$CCA(t+i) = 0$
 $Z2(t+i) = \max(0, Z1(t+i))$

No capital cost allowance with offset:

$CCA(t+i) = 0$
 $Z2(t+i) = Z1(t+i)$

Capital cost allowance without offset:

$$CCA(t+i) = \max(0, \min[Z1(t+i), DEP(t+i)])$$

$$Z2(t+i) = \max[0, Z1(t+i) - CCA(t+i)]$$

Capital cost allowance with offset:

$$CCA(t+i) = DEP(t+i)$$

$$Z2(t+i) = Z1(t+i) - CCA(t+i)$$

Proceeds from Sale:

$$L^e(t+T) + B^e(t+T) = \text{total property value}$$

$$MB(t+T) = \text{mortgage balance}$$

$$= (1 - EQ(t))V(t) - \sum_1 P(t+i)$$

$$PROC(t+T) = \text{proceeds}$$

$$= (1 - \sigma)[L^e(t+T) + B^e(t+T)] - MB(t+T)$$

Tax on Sale:

$$W(t+T) = \text{capital gains after expenses}$$

$$= (1 - \sigma)[L^e(t+T) + B^e(t+T)] - [B(t) + L(t)]$$

$$NDEP(t+T) = \text{undepreciated balance}$$

$$RECAP = \text{recapture} = r[B(t) - NDEP(t+T)]$$

$$TTAX(t+T) = \text{total tax}$$

$$= \max[0, RECAP + \hat{r}W(t+T)] \quad \text{with no offset}$$

$$= RECAP + \hat{r}W(t+T) \quad \text{with offset}$$

Present Value:

$$PDV(t) = \sum_1 ATCF(t+i)/(1+d(t+i)) \\ + [PROC(t+T) - TTAX(t+T)]/(1+d(t+T))$$

V. Limited Dividend Program

$$EQ(t) = \text{set equity at 5\% less than above}$$

$$MR(t) = \text{set mortgage rate at 2\% below above with 50 yr.} \\ \text{amortization period}$$

$TC(t+1)$ = total cost in period $t+1$

$$= OC(t+1) + I(t+1) + P(t+1) + \epsilon(EQ(t))V(t)$$

$R(t+1)$ = first yr. rent set to yield ϵ on equity

$$= TC(t+1)$$

$R(t+i) - R(t+i-1)$ = rent changes ($i > 2$) set to cover cost changes

$$= OC(t+i) - OC(t+i-1)$$

VI. Assisted Rental Programs

ARP 1975 (non-repayable, non-taxable grants)

$ARPL5(t+i)$ = ARP 1975 loan in $t+i$

$$TC(t+1) = OC(t+1) + I(t+1) + P(t+1) + \epsilon(EQ(t))V(t)$$

If $TC(t+1) < R(t+1)$ set $ARPL5(t+1) = 0$

otherwise set $ARPL5(t+1) = \min[900, TC(t+1) - R(t+1)]$

$\Delta ARPL5$ = loan decrease between periods

$$= (ARPL5(t+1))/10$$

$$ARPL5(t+i) = \max[0, ARPL5(t+i-1) - \Delta ARPL5]$$

$R5(t+i)$ = rent at $t+i$ under ARP 1975

$$= \min[R^e(t+i), R5(t+i-1) + OC(t+i) - OC(t+i-1) + \Delta ARPL5]$$

$R^e(t+i)$ = $R5(t+i-1)$ times one plus the percentage change in market rents between $t+i-1$ and $t+i$

$ATCFA5(t+i)$ = after tax cash flow with ARP 1975

$$= ATCF(t+i) + ARPL5(t+i)$$

ARP 1976 (repayable, interest free loans)

$ARPL6(t+i)$ = ARP 1976 loan in $t+i$

$$TC(t+1) = OC(t+1) + I(t+1) + P(t+1) + \epsilon(EQ(t))V(t)$$

If $TC(t+1) < R(t+1)$ set $ARPL6(t+1) = 0$

otherwise set $ARPL6(t+1) = \min[1200, TC(t+1) - R(t+1)]$

$TARPL6(t+i)$ = cumulative ARP loans at $t+i$

set $TARPL6(t) = 0$

$TARPL6(t+i) = TARPL6(t+i-1) + ARPL6(t+i)$

$\Delta ARPL6$ = loan decrease between periods (a lower bound)

$$= (ARPL6(t+1))/10$$

$\Delta(t+i)$ = actual loan decrease adjusted for return on equity

$$= \max[\Delta ARPL6, R(t+i) - R(t+i-1) - OC(t+i) + OC(t+i-1)]$$

$$ARPL6(t+i) = \max[0, ARPL6(t+i-1) - \Delta(t+i)]$$

$ATCFA6(t+i)$ = after tax cash flow with ARP 1976

$$= ATCF(t+i) + ARPL6(t+i)$$

Subtract $TARPL6(t+T)$ from the after tax sales proceeds

ARP 1977 Same as ARP 1976 except that:

$$ARPL7(t+1) = \min[Z, TC(t+1) - R(t+1)] \quad \text{where } 900 < Z < 1200$$

Z = a weighted average of 1200 and 900, where the weights used are the relative weights for Toronto plus Vancouver versus the rest of Canada that are used by Stats Canada in calculating national land and housing price indices (see Appendix B).

ARP 1978 (payment reduction loan)

$ARPL8(t+i)$ = ARP 1978 loan in $t+i$

$$TC(t+1) = OC(t+1) + I(t+1) + P(t+1) + \epsilon(EQ(t))V(t)$$

If $TC(t+1) < R(t+1)$ set $ARPL8(t+1) = 0$

otherwise set $ARPL8(t+1) = \min[0.027(1 - EQ(t))V(t), TC(t+1) - R(t+1)]$

TARPL8(t+i) = cumulative ARP loans at t+i

set TARPL8(t) = 0

TARPL8(t+i) = TARPL8(t+i-1) + ARPL8(t+i)

TARPI8(t+i) = total ARP interest owed at the end of t+T

TARPI8(t+i) = TARPI8(t+i-1) + ARPL8(t+i)(1+MR(t))^{T-i}

Z3(t+i) = 5% of previous year's interest + principal minus
previous year's payment reduction loan (PRL)

= .05[(I(t+i-1)+P(t+i-1)-ARPL8(t+i-1))]

ARPL8(t+i) = max[0, ARPL8(t+i-1)-Z3(t+i)]

ATCFA8(t+i) = after tax cash flow with ARP 1978

= ATCF(t+i) + ARPL8(t+i)

Subtract TARPL8(t+T) and (1-r)TARPI8(t+T) from sales proceeds

VII. Canada Rental Supply Plan (repayable interest free loan)

CRSP(t) = CRSP loan at time t

= max[10500, .8V(t)-(1-EQ(t))V(t)]

Subtract CRSP(t) from after tax sales proceeds

Table 1
Approval Data

<u>Program</u>	<u>Year</u>	<u>1st Mortgage/unit</u>	<u>Estimated Unit Value</u>
LD	1968	11733	12350
	1969	12120	12760
	1970	12645	13310
	1971	12495	13150
	1972	12820	13500
	1973	13770	14500
	1974	16984	17880
	1975	22600	23790
ARP	1975	20825	23140
	1976	23085	25650
	1977	27900	31000
	1978	30230	33590
CRSP	1982	25350	45270
	1983	34870	62270
	1984	39565	70650
	1985	38460	68680

Table 2

Capital Budget - Loans and Investments (\$ millions)

Note: Non-Budgetary Funds = funds loaned in each period
 Budgetary Expenditures = grants, contributions, subsidies and cost of capital (owed to Treasury Board) as well as administrative expenses and loan losses.

Limited Dividend

Non-Budgetary Funds

1970	241.2	1975	235.2
1971	231.9	1976	9.2
1972	94.7	1977	5.6
1973	59.5	1978	1.9
1974	74.5		

Assisted Rental Program

Non-Budgetary Funds*

1976	137.5	1981	35.6	1987	9.4
1977	320.8	1982	47.2		
1978	96.2	1983	29.7		
1979	0.4	1984	40.2		
1980	-				

Budgetary Expenditures

1975	0.2	1980	25.0	1985	35.3
1976	2.6	1981	29.7	1986	47.4
1977	9.9	1982	32.3	1987	51.9
1978	17.7	1983	36.8		
1979	19.5	1984	46.4		

Canada Rental Supply Program

Non-Budgetary Funds

1982	117.9	1985	5.1
1983	110.1		
1984	24.3		

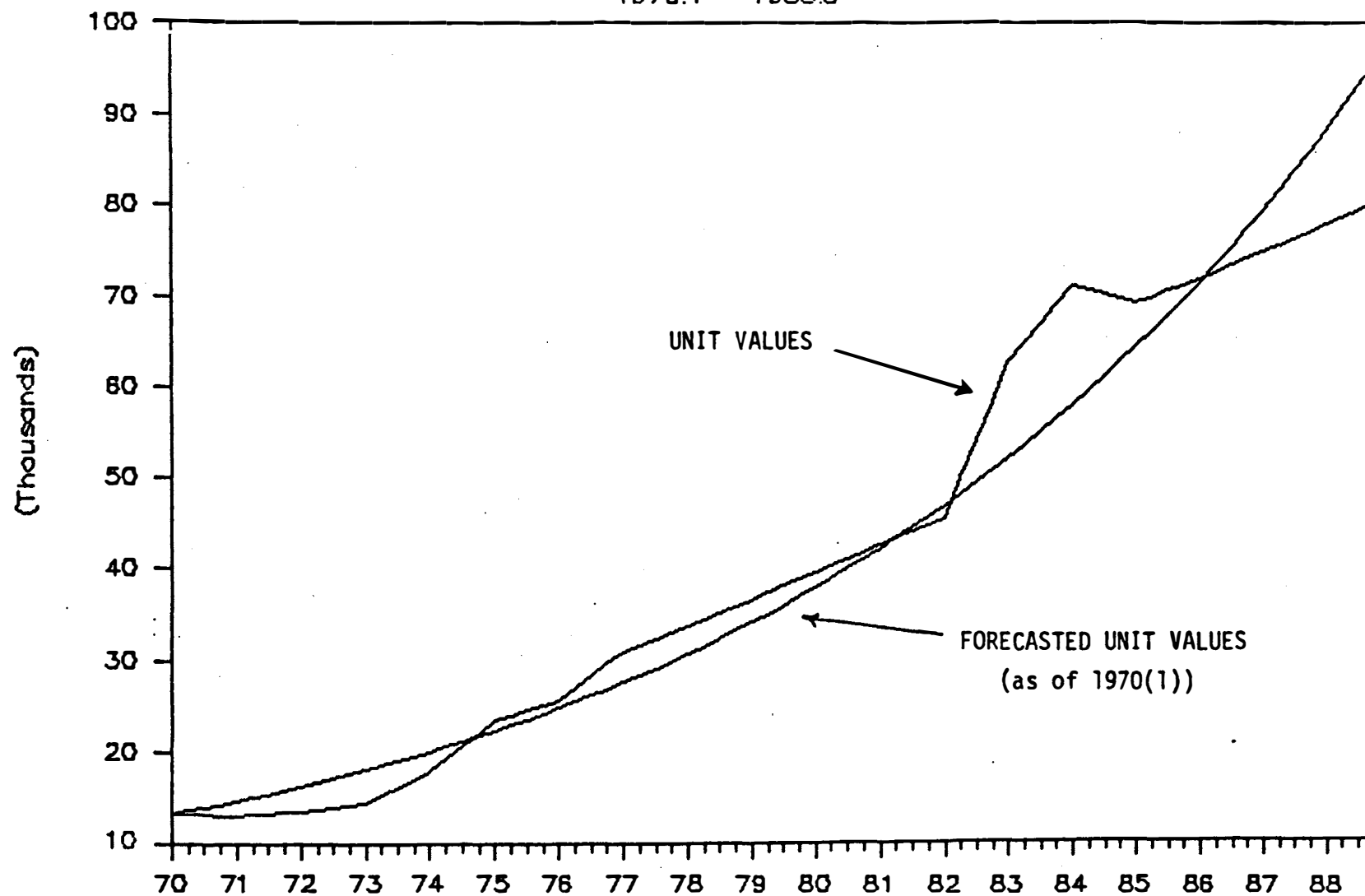
Budgetary Expenditures

1982	1.2	1985	15.4
1983	5.4	1986	23.2
1984	15.1	1987	24.1

*The 1981-87 funds represent supplementary assistance to clients.

FIGURE 1a

1970:1 - 1988:3



MORTGAGE RATE (IN PERCENT)

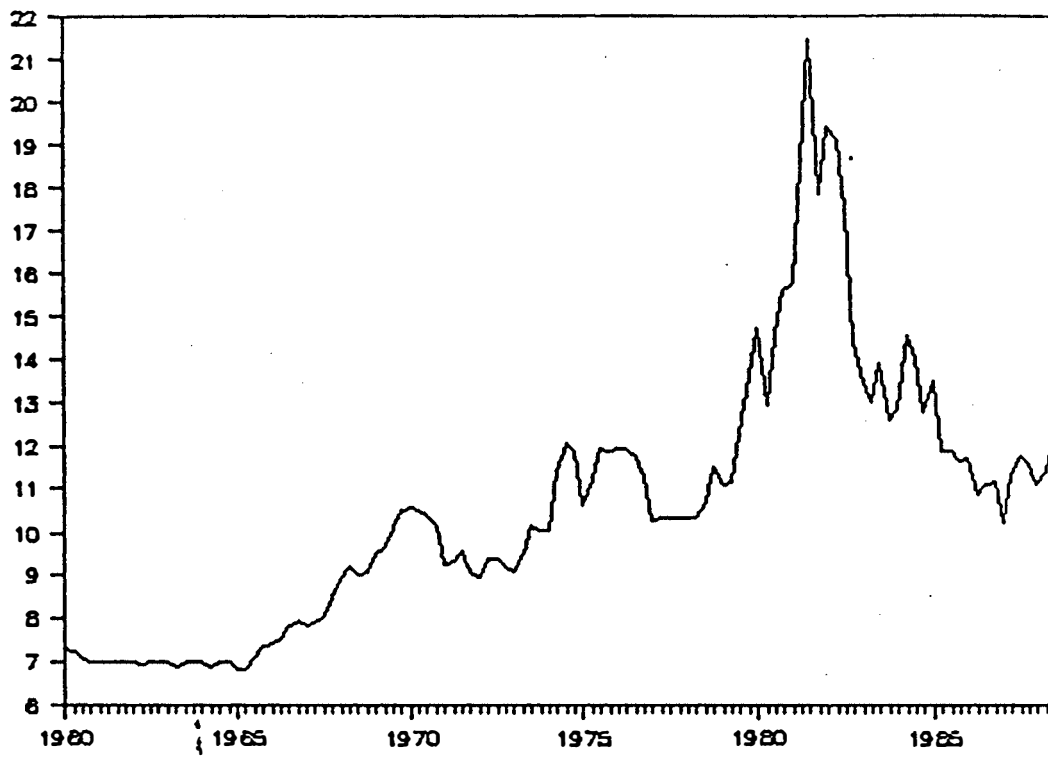


FIGURE 1b

FIGURE 2

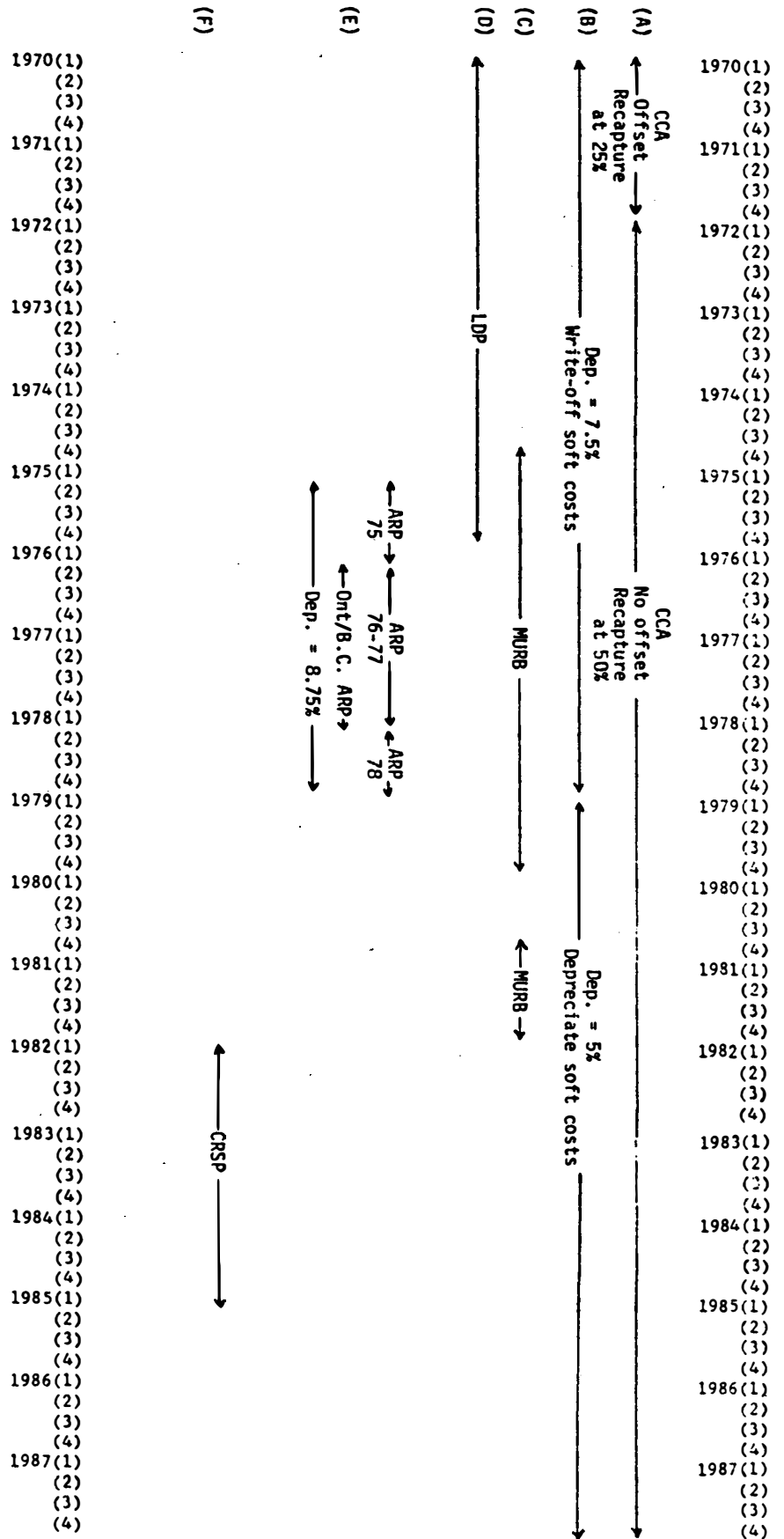


TABLE 3

PRESENT VALUE OF SUBSIDY PROGRAMS

DATE	BASE	MURB	LDP	ARP	ONTARP	BCARP	CRSP	ARP + MURB	ONTARP + MURB	BCARP + MURB
70	12211	12211	13317	12211	12211	12211	12211	12211	12211	12211
70	11849	11849	13038	11849	11849	11849	11849	11849	11849	11849
70	11909	11909	13020	11909	11909	11909	11909	11909	11909	11909
70	11986	11986	13005	11986	11986	11986	11986	11986	11986	11986
71	12448	12448	13085	12448	12448	12448	12448	12448	12448	12448
71	12754	12754	13342	12754	12754	12754	12754	12754	12754	12754
71	12713	12713	13397	12713	12713	12713	12713	12713	12713	12713
71	13060	13060	13552	13060	13060	13060	13060	13060	13060	13060
72	11503	11503	12158	11503	11503	11503	11503	11503	11503	11503
72	11789	11789	12554	11789	11789	11789	11789	11789	11789	11789
72	11987	11987	12774	11987	11987	11987	11987	11987	11987	11987
72	12303	12303	13020	12303	12303	12303	12303	12303	12303	12303
73	12615	12615	13266	12615	12615	12615	12615	12615	12615	12615
73	14315	14315	14843	14315	14315	14315	14315	14315	14315	14315
73	14503	14503	15483	14503	14503	14503	14503	14503	14503	14503
73	15252	16363	16241	15252	15252	15252	15252	16363	16363	16363
74	15893	17065	16977	15893	15893	15893	15893	17065	17065	17065
74	16960	18411	18833	16960	16960	16960	16960	18411	18411	18411
74	17162	18915	19916	17162	17162	17162	17162	18915	18915	18915
74	18293	20131	21139	18293	18293	18293	18293	20131	20131	20131
75	20798	22429	22591	20798	20798	20798	20798	22429	22429	22429
75	18473	20430	21650	22609	18486	18486	18473	24723	20600	20600
75	17873	20140	21985	21996	17882	17882	17873	24429	20315	20315
75	18314	20613	22464	22437	18322	18322	18314	24907	20791	20791
76	18670	21026	18670	22794	18679	18679	18670	25323	21208	21208
76	21390	23726	21390	23276	24719	25977	21390	25791	27234	28492
76	22560	24955	22560	24388	25847	27210	22560	26970	28429	29792
76	24287	26611	24287	26067	27459	28672	24287	28581	29973	31186
77	26925	29024	26925	28666	29604	30532	26925	30950	31888	32816
77	25706	27959	25706	27045	27979	29837	25706	29493	30428	32286
77	26251	28542	26251	27572	28492	30429	26251	30063	30983	32920
77	26734	29076	26734	28037	28943	30998	26734	30583	31489	33543
78	27229	29618	27229	28515	29410	31569	27229	31112	32007	34166
78	27991	30420	27991	29166	28020	28020	27991	31807	30662	30662
78	27895	30513	27895	29190	27921	27921	27895	32030	30760	30760
78	26733	29819	26733	28299	26749	26749	26733	31623	30072	30072
79	28082	29038	28082	28082	28082	28082	28082	29038	29038	29038
79	28508	29510	28508	28508	28508	28508	28508	29510	29510	29510
79	26666	28325	26666	26666	26666	26666	26666	28325	28325	28325
79	24014	26714	24014	24014	24014	24014	24014	26714	26714	26714
80	21630	21630	21630	21630	21630	21630	21630	21630	21630	21630
80	26494	26494	26494	26494	26494	26494	26494	26494	26494	26494
80	22872	22872	22872	22872	22872	22872	22872	22872	22872	22872
80	20215	25203	20215	20215	20215	20215	20215	25203	25203	25203
81	20108	25364	20108	20108	20108	20108	20108	25364	25364	25364
81	11892	20984	11892	11892	11892	11892	11892	20984	20984	20984

TABLE 3 (cont.)

PRESENT VALUE OF SUBSIDY PROGRAMS

[illegible]

FIGURE 3

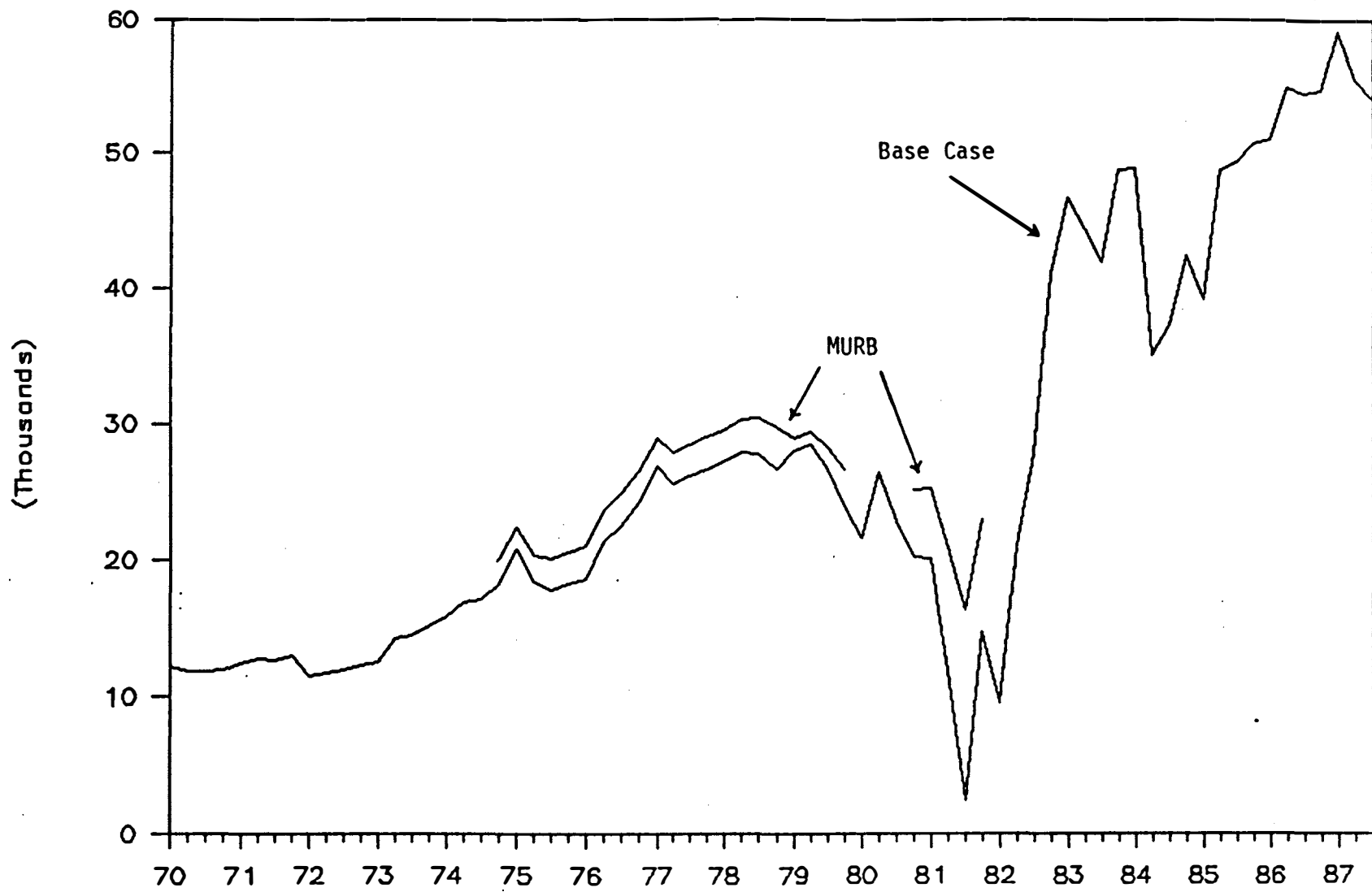


FIGURE 4

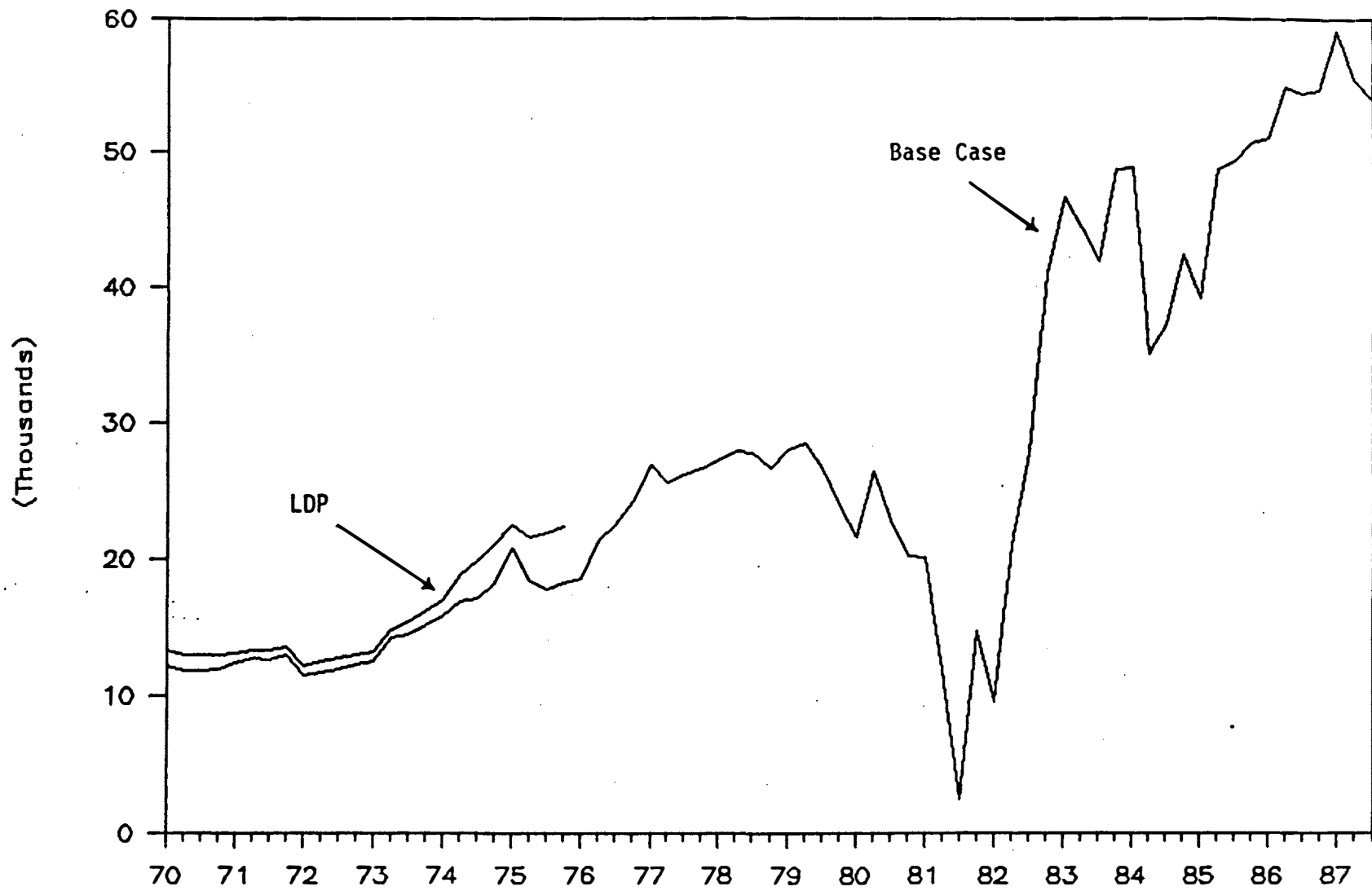


FIGURE 5

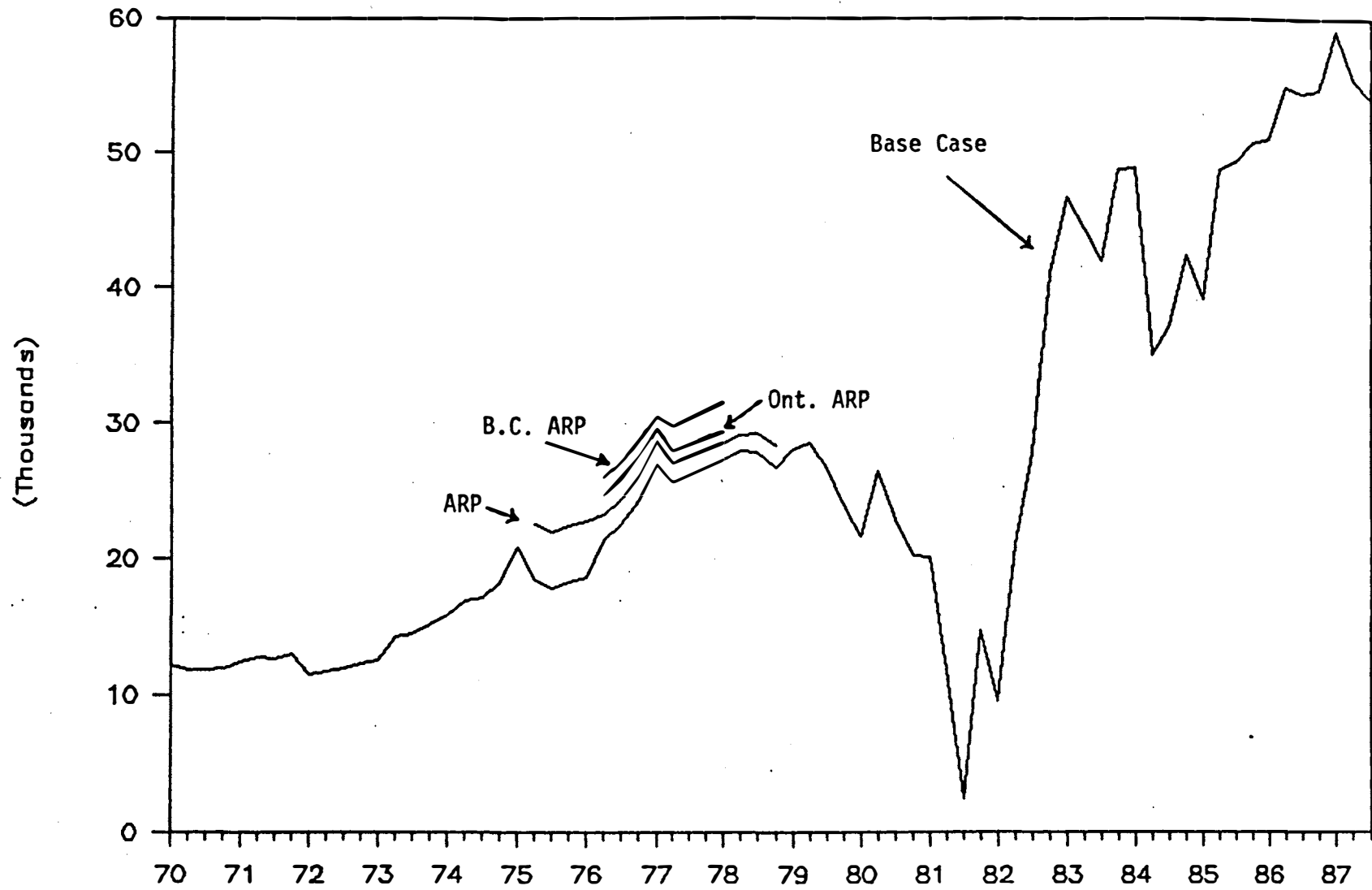


FIGURE 6

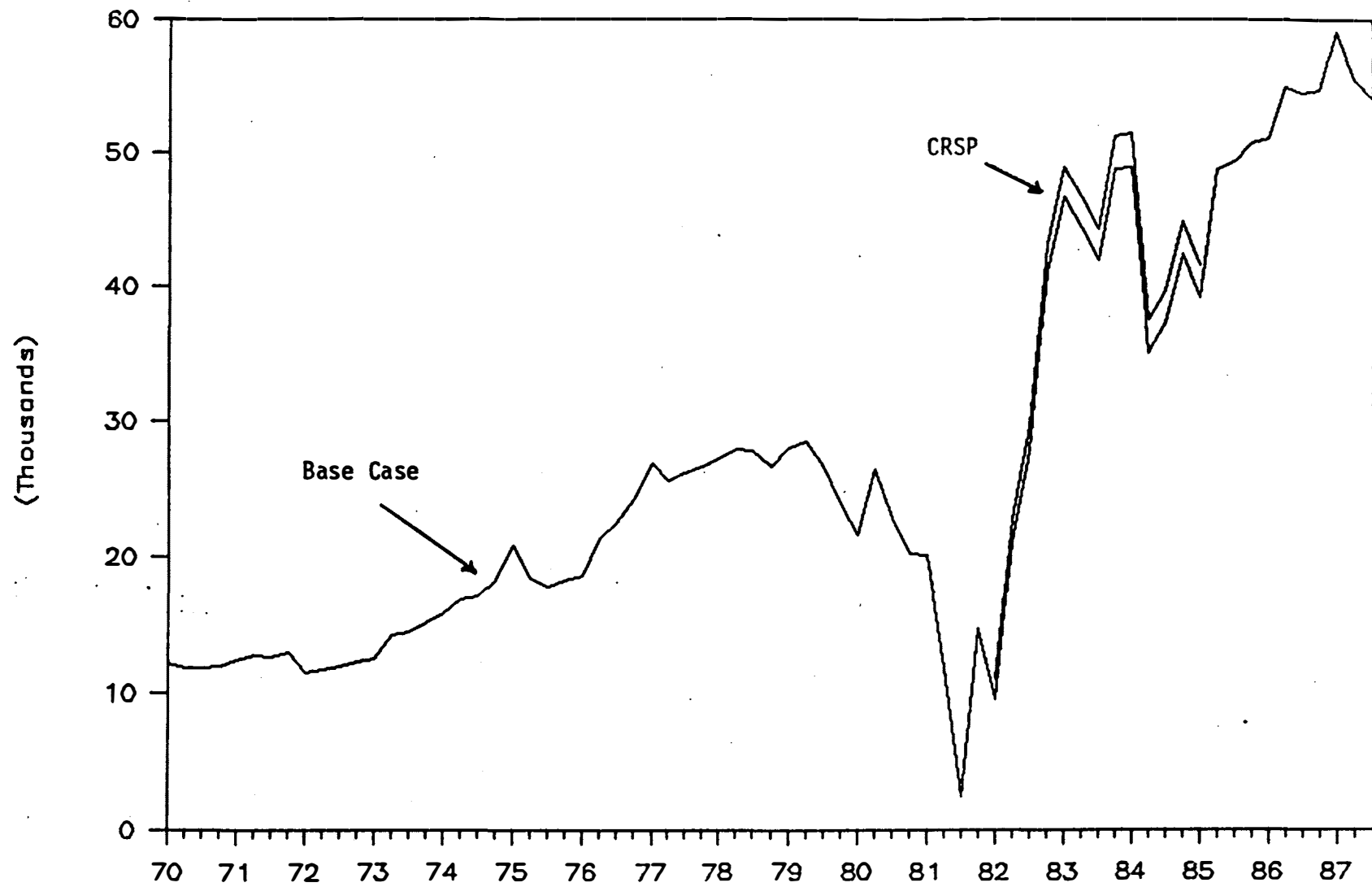


FIGURE 7

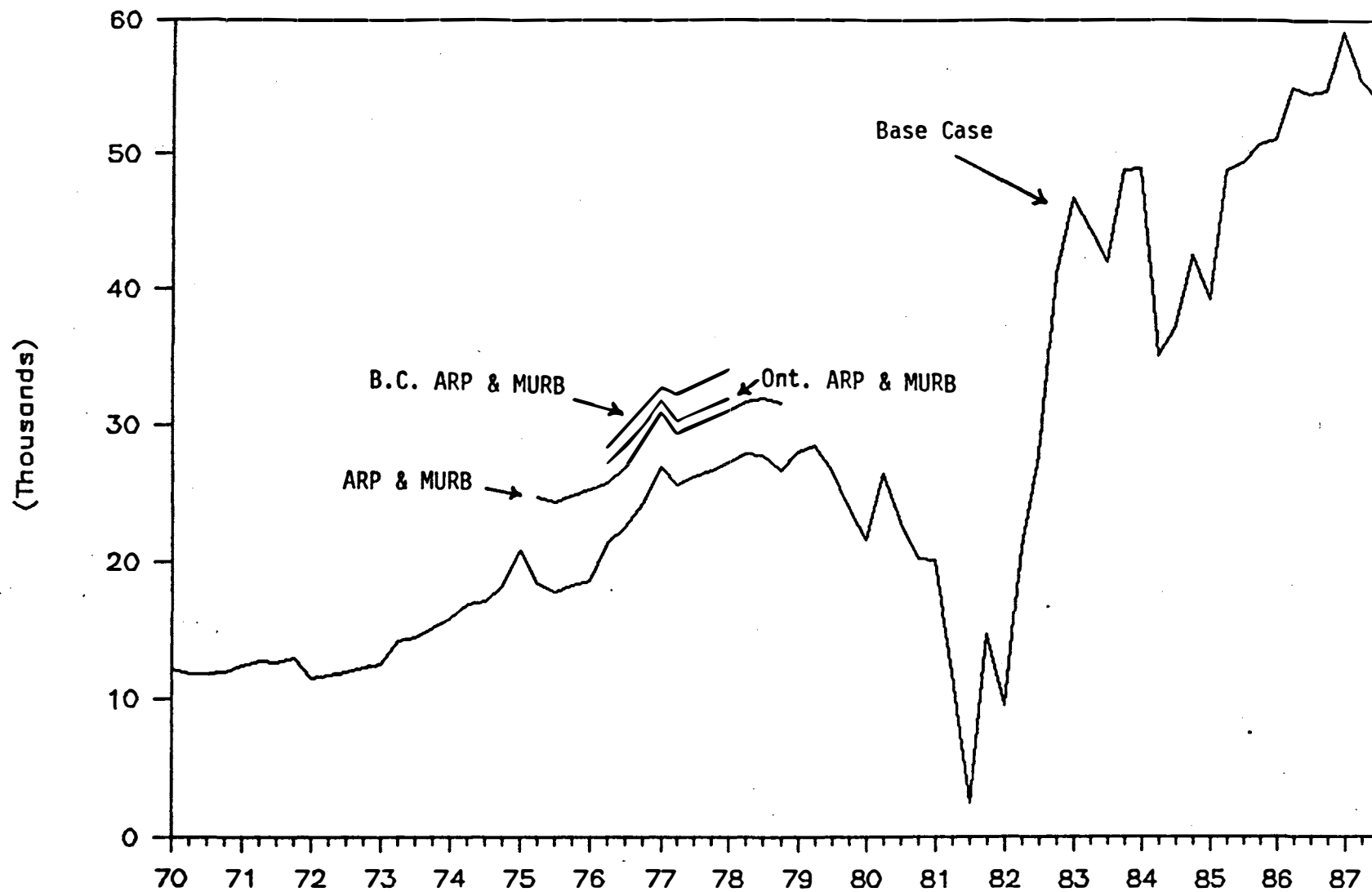


Table 4

Calculation of MURB Subsidy Costs (\$ thousands)

Year	1975	1976	1977	1978	1979	1981
Starts	8517	35219	82265	80089	76550	61500
Costs	23140	25650	31000	33590	36277	42314
1975	451					451
1976	417	2067				2935
1977	386	1912	5833			11066
1978	357	1768	5396	6154		24741
1979	330	1636	4991	5692	6352	43742
1980	305	1513	4617	5266	5876	61319
1981	282	1400	4270	4871	5435	5953 83527
1882	261	1295	3950	4505	5027	5507 104072
1983	241	1197	3654	4167	4650	5094 123075
1984	223	1108	3380	3855	4301	4712 140654
1985		1025	3126	3566	3979	4358 153514
1986			2892	3298	3680	4031 152496
1987				3051	3404	3729 120569
1988					3149	3449 82744
Total	3194	14919	42111	44423		

Table 5

Estimated MURB Subsidy Costs (\$ Millions)

	<u>PV</u>	<u>Alt</u>
1975	1.42	0.45
1976	7.31	2.94
1977	21.77	11.07
1978	41.51	24.74
1979	56.11	43.74
1980	63.97	61.32
1981	102.15	83.53
1982	125.87	104.07
1983	125.87	123.08
1984	125.77	140.65
1985	124.45	153.51
1986	118.55	152.50
1987	104.09	120.57
1988	84.37	82.74

Table 6

Estimated LD Subsidy Costs (\$ millions)

	<u>PV</u>	<u>Alt</u>	<u>CHS</u>
1971	2.60	6.76	4.64
1972	3.25	9.25	6.53
1973	3.70	10.87	7.72
1974	4.14	11.88	8.91
1975	6.38	15.28	13.61
1976	8.0	17.13	13.79
1977	8.0	17.13	13.91
1978	8.0	17.13	13.95
1979	8.0	17.13	13.95
	.	.	.
	.	.	.
	.	.	.
1988	8.0	17.13	13.95

Table 7

Estimated ARP Subsidy Costs (\$ millions)

	<u>PV</u>	<u>PVF</u>	<u>Alt</u>	<u>CHS</u>
1975	4.61	4.61	1.6	0.2
1976	14.41	13.61	5.47	2.6
1977	24.70	20.59	13.72	9.9
1978	30.89	24.83	22.31	17.7
1979	31.85	25.8	29.86	19.5
1980	31.85	25.8	36.41	25.0
1981	31.85	25.8	41.97	29.7
1892	31.85	25.8	46.56	32.3
1983	31.85	25.8	50.12	36.8
1984	31.85	25.8	52.64	46.4
1985	27.23	21.18	54.21	35.2
1986	17.44	12.24	41.67	47.4
1987	7.5	5.2	15.42	51.9

Table 8

Estimated CRSP Subsidy Costs (\$ millions)

	<u>PV</u>	<u>Alt</u>	<u>CHS</u>
1982	1.58	3.56	1.2
1983	3.33	9.22	5.4
1984	4.78	12.52	15.1
1985	5.31	13.7	15.4
1986	5.31	13.7	23.2
1987	5.31	13.7	24.1

May 16, 1989

Appendix B

**A FRAMEWORK FOR THE ANALYSIS
OF RENTAL HOUSING PROGRAMS**

Institute for Policy Analysis

University of Toronto

CONTENTS

1. INTRODUCTION	1
2. A STOCK-ADJUSTMENT MODEL OF THE RENTAL HOUSING MARKET	2
(i) The Model	3
(ii) Static Expectations	6
(iii) Rational Expectations	12
(iv) Extensions: Rent Control	17
3. A TAXONOMY OF RENTAL HOUSING PROGRAMS	21
(i) Terms of Assistance	22
(ii) Duration	24
(iii) Availability	25
(iv) Rent Restrictions	28

1. INTRODUCTION

The federal government has used a diversity of rental housing programs over the last twenty-five years, including the Limited Dividend Program (LD), the Assisted Rental Program (ARP), the Multiple Unit Residential Buildings Program (MURB), and the Canada Rental Supply Plan (CRSP). The government seeks to understand the effects of these programs at a micro level on housing starts, rents, vacancy rates and house prices and at a macro level on the cyclical stability of residential construction, on employment and on government revenues and expenditures. This Appendix is part of a larger project which analyses the micro and macro effects of these programs.

This Appendix provides a framework for the analysis of rental housing programs. More specifically, the Appendix has two purposes. First, it sets out in a relatively non-technical way the micro-economic model of the rental housing market which underlies the analysis of the larger project. And second, it identifies those factors which affect the total net rental completions caused by the rental housing programs and which affect the time path of these net completions. It discusses the channels through which these factors influence the rental housing market.

Section 2 outlines a stock-adjustment model of a rental housing market. The section draws out those aspects of a rental market which most shape the short-run and long-run response to any rental assistance program; in particular the distinction between short-run and long-run elasticity of supply, the role of expectations and whether or not the market is subject to rent controls. Section 3 provides a taxonomy of the types of rental housing programs which a government could use. The

taxonomy highlights those features in the design of a rental assistance program which most shape the short-run and long-run response; in particular the terms of assistance, whether the program is permanent or temporary, whether there is a limited budget for assistance, and whether there are restrictions on the rents which can be charged on assisted units. Thus in the analysis of what factors shape the effects of rental housing programs, Section 2 focuses on the nature of the rental market and Section 3 focuses on aspects of the design of housing assistance programs.

2. A STOCK-ADJUSTMENT MODEL OF THE RENTAL HOUSING MARKET

The model of a rental housing market appropriate for this project is a stock-adjustment model, because the focus of interest is on government assistance to investors, on how investors capitalize the value of this assistance and on its effects on rents and new construction over time. This stock-adjustment model for a rental market without rent controls is set out graphically in Figure One. (It is widely used in housing analysis and more detailed presentations can be found in Smith (1974), (1987), and Fallis (1985)).

Ideally for the analysis of housing markets, a model should be used which articulates separate rental and ownership markets, allowing households to move between sectors (i.e., providing analysis of tenure choice) and allowing developer/builders to produce new housing in either sector. The ideal model would also make household formation endogenous. A household is a group of people (or one person) sharing a dwelling unit.

The decision to form a household - for example, when a young person decides to leave his/her parents' dwelling and set up on his/her own - is influenced by the price of rental housing. The total demand for housing is influenced by total population and also by how many and what type of households are formed within the population. However, as starting point for the analysis of rental housing programs, the ownership market and household formation cannot be formally modelled. The rental market will be considered alone.

(i) The Model

The demand for rental housing units is DD^1 . The supply of rental housing units in any one period is perfectly inelastic, SS . The supply of rental units comes from existing buildings, and in the initial period is S_0 . The market for rental housing units determines the rent per unit R_0 . This simple model does not analyze vacancies or the relationship between rents and vacancies. (The econometric work of the larger project does include a relationship between rents and vacancies.)

When investors purchase a dwelling unit, they are in effect purchasing the after-tax cash flows from the unit; flows that occur in the current period and in future periods. Given the current rent per dwelling unit, current operating costs, current mortgage financing arrangements, the current tax system and expectations about future values of these, investors compute current and expected future after-tax cash flows. The price of a dwelling unit, V_0 , is the present value of the after-tax cash flows. To illustrate, assume a simple world in which there is depreciation at rate δ , no tax system, no inflation, no

change in future demand, a constant real interest rate r , no operating costs on the dwelling unit, and further, assume all investments including a financial asset earn a return r and are equally risky and liquid. When the rental market is in equilibrium there will be no change over time in V , R or S . Investors believe, correctly, that R_0 will persist ($R_0 = R_1 = R_2 \dots$) and the price of a dwelling unit would just equal $R_0/(r+\delta)$ as in (1). The determination

$$V_0 = \frac{R_0}{1+r} + \frac{R_1(1-\delta)}{(1+r)^2} + \frac{R_2(1-\delta)^2}{(1+r)^3} + \dots$$

(1)

$$= \frac{R_0}{r+\delta} \quad (\text{if } R_0 = R_1 = R_2 \dots)$$

of dwelling unit price with explicit specification of expected rents, operating costs, mortgage financing and taxes is set out and discussed in a related Appendix to this study. This process of taking the present value of future cash flows plays a critical role in analyzing rental programs because a government program alters current and future cash flows on a unit and therefore alters the amount investors are willing to pay for the unit.

Because new construction in any one period is such a small percentage of total stock, any level of new construction will alter rents only slightly. This is especially true for a quarterly model, as is used in this project. Therefore, it is assumed that in any one period there is a perfectly elastic demand for new stock, II, on Figure One. The prices of inputs (labour, materials and land), the elasticity of their supply to the rental housing sector, and the technology of producing

housing stock determine the new construction curve for any one period, CC.

An essential feature of the stock-adjustment model is that CC is upward sloping. In the short run to build more dwelling units requires that more labour, materials and land be drawn into the residential construction sector. These factors are not perfectly elastically supplied and thus as the construction industry expands, factor prices rise. This is especially the case for land because the flow supply of land is restricted by the municipal approval process.

The level of construction in any one period C is determined by the intersection of the investor demand curve II and the new construction curve, CC. (Investors compare the value they place on the after-tax cash flows, V , with the cost of construction. Construction occurs up to the point where V equals construction costs).

The new housing units constructed in any period, for example C_0 , are added to the existing stock at the beginning of the next period. If there were no depreciation, S_1 would equal $S_0 + C_0$. If there were depreciation at rate δ , S_1 would equal $S_0 - \delta S_0 + C_0$. In the next period, there is again a perfectly inelastic supply of units; and the rent per unit is determined by the intersection of the demand for units, DD, and supply of units, S_1 .

In this stock-adjustment model there is a sharp distinction between the short-run response of new construction (reflected in the CC curve) and the long-run response. The short-run elasticity is quite small, relative to the long-run elasticity. Housing markets take a long time to

adjust. (For further discussion see Poterba (1984) and Topel and Rosen (1988)).

When the rental market is in equilibrium, R and V will be constant and (assuming no growth in demand) new construction in each period will just equal depreciation ($C_0 = \delta S_0$). The level of construction and total number of dwelling units will be constant over time.

To get an idea of the dynamics of the model, consider an outward shift in the demand function to $D'D'$. This shift is permanent, occurs at the beginning of period one and was unanticipated. Rents would increase to R_1 as in Figure Two. (With a vacancy rate model, this adjustment would be lagged.) The housing programs analyzed do not stimulate demand but it is useful to explore the dynamics of the model with a familiar example before turning to the rental housing programs.

(ii) Static Expectations

The usual approach in this model is to assume investors believe that R_1 will persist forever and so are willing to bid $V_1 = R_1/(r + \delta)$ for a dwelling unit. Such expectations can be called static or myopic because the current rent is expected to continue into the future. During the first period, there will be new construction of C_1 which is added to the existing housing stock; and at the end of the period there are $S_0 + C_1 - \delta S_0$ units of housing stock. At the beginning of the second period, rents fall to R_2 . Again investors believe this will continue into the future when valuing housing stock; the price of stock falls to $R_2/(r + \delta)$ and construction falls to C_2 . Thus after their

initial increases, rents, stock price and construction levels then fall over time until a new equilibrium is restored. The new equilibrium will have a higher rent per dwelling unit, a higher value per dwelling unit and a larger housing stock than the original equilibrium. Annual construction will be somewhat higher and just replace depreciation each year.

Figure Two gives a qualitative idea of the time path of new construction, but a better idea can be conveyed by a small simulation model. The model which has a linear demand function DD and a linear construction supply function CC is set out formally in endnote 2. The demand function has a price elasticity of -1 and the construction supply curve has an elasticity of 3 (in the neighbourhood of the initial equilibrium). The price elasticity of demand is consistent with, or slightly greater than, most empirical findings. The supply elasticity is higher than many estimates but is consistent with the recent findings of Topel and Rosen (1988). (For a survey of empirical findings see Olsen 1986.) The depreciation rate is assumed to be $.01$ and the real interest rate to be $.05$. At the initial equilibrium, rental stock is 3 million units, the annual unit rent is \$4800 (\$400 per month) and the unit value is \$80,000. These seem reasonable values for Canada in the early 1980s. Annual construction is 30,000 units, which just replaces depreciation. (Actual annual levels have been above this because of demand growth; but this model has no demand growth.)

Now consider a permanent shift up in the demand curve such that rent rises to \$5400, a 12.5 per cent increase. With static expectations, investors believe this new rent will persist and therefore value each

unit at \$90,000 which gives rise to construction of 41,250 units. The 12.5 per cent price rise increases construction by 11,250 units or 37.8 per cent in period one. This is a net addition over depreciation and next period the housing stock is 3,011,250. The stock is 0.375 per cent larger. Thus even though the construction supply elasticity is 3; the one-period supply elasticity is very, very low (0.03). Even a large percentage jump in construction does not yield a large percentage increase in total stock. Housing markets adjust very, very slowly.

The time path of the variables of this simulation model are reported in Table One for static expectations. Rents jump from 48 to 54 and then gradually return to a new long run equilibrium level of 49.5. Values jump from 800 to 900 and gradually return to 825. Construction jumps to 4.125 and gradually declines, until at the new equilibrium it just replaces depreciation. Housing stock gradually rises to its new equilibrium. But the adjustment is very slow. After 8 periods only 28 percent of the gap has been closed between period one unit value and the final equilibrium. Housing stock similarly has only closed 28 percent of the gap.

This model can be used to analyze the influence of government housing programs, although the sorts of rental programs examined in this project do not stimulate demand as in Figure Two. (A government program such as a housing allowance to tenants would stimulate demand.) Most of the programs of this project either offer more attractive mortgage financing than is available in the private market or change the income tax treatment of the rental housing sector. These programs alter the value investors place on owning a rental unit. If investors have static

expectations, they assume that the current and future rents which they expected prior to the assistance program will be unchanged and so calculate V based on $R_0 = R_1 = R_2 = \dots$. The attractive mortgage financing or tax changes raise the expected after-tax cash flows which result from the expected rents and therefore the price investors are willing to pay for a dwelling unit rises. The II curve shifts up and new construction increases. Next period housing stock has increased, rents are lower, but again if investors have static expectations and they assume this lower R will persist.

In equation (1), it was implicitly assumed that the mortgage rate of interest, rm , was equal to the return on a financial asset, r . If the financing of the purchase of a dwelling unit were explicitly introduced, equation (1) would be rewritten as (2), assuming that the mortgage is simply a

$$V_0 = M + \frac{R_0 - M \cdot rm}{1+r} + \frac{R_1(1-\delta) - M \cdot rm}{(1+r)^2} + \frac{R_2(1-\delta)^2 - M \cdot rm}{(1+r)^3} + \dots \quad (2)$$

$$= M + \frac{R_0}{r+\delta} - \frac{M \cdot rm}{r} \quad (\text{if } R_0 = R_1 = \dots)$$

perpetual loan on which interest payments must be made and where M is the mortgage principal. If $r = rm$, then equation (2) reduces to equation (1).

Suppose that the government were to make available mortgage loans at less than the private market rate; for the sake of clarity, assume the same sized mortgage was used, with and without assistance. It is obvious from (2) that reducing rm , raises the annual cash flows and therefore

raises V . This argument assumes that expectations about future rents are unchanged by the program, i.e., investors have static expectations.

The terms of assistance of an actual rental program are more complex usually than a reduction in r_m , but the principle in equation (2) remains. The assistance alters the after-tax cash flows and raises V .

Consider a specific example using the simulation model outlined above. The government initiates a rental housing assistance program at the beginning of period one, which makes mortgage loans available to all investors at less than market interest rates. The program is permanent and was unanticipated. The market mortgage interest rate is 0.05 (there is no inflation and r is equal to r_m), and government loans are available at a rate of 0.0403. This is about a 19 per cent reduction in the interest rate. The original loan-to-value ratio was assumed to be 0.75 and this ratio remains after assistance is available. The interest rate reduction of this government program and the loan-to-value ratio were selected so that the government housing program resulted in the same long-run equilibrium housing stock and unit value as the demand shift example of Table One.

Under the rental assistance program with static expectations, rent in period one remains at \$4,800, value rises from \$80,000 to \$91,034 and construction rises from 30,000 to 42,413 units. This is a net addition to stock over depreciation and period two stock is 3,012,413. Rents fall in period two to \$4,780. Again assuming static expectations, value falls to \$90,660 in period two and construction falls to 41,990.

The full time path of adjustment to the rental assistance program under static expectations is set out in Table Two. Values, construction

and stock adjust as they did when the demand shifted. Value rises to \$91,034 in period one, then slowly falls to a new long-run equilibrium which is higher than the original equilibrium. Annual construction rises to 42,413 then slowly falls to a new long run equilibrium which is higher than the original equilibrium. Stock is unchanged in period one but then rises to a level which is approximately 9.1 per cent larger than the original equilibrium stock.

In contrast to the demand shift in Table One, rents after a mortgage assistance program are at first unchanged. Then they gradually fall as new stock is added. The new long-run equilibrium rent is \$4,350, well below the original equilibrium.

In both the above examples - the demand shift and the government housing program - investors were assumed to have static expectations. However this assumption of static expectations is unrealistic. Returning to Figure Two and the numerical example of a demand shift, investors would foresee that the jump in rents to 54 would not persist. They would recognize that the increased construction induced by the demand shift (or government housing program) would cause rents to fall in the future. And furthermore, if investors held static expectations, their investments would be systematically unprofitable. Consider an investor owning a unit for one period. Costs for the period are foregone interest on equity plus mortgage interest plus depreciation. Assuming the mortgage and financial rate of interest are the same, costs are $V_1(r + \delta)$. Returns to the one-period investment are rents plus capital gains (or in this case rents plus capital loss because values fall over time). Returns are $R_1 + (V_2 - V_1)$. With static expectations, investors

assume R_1 will persist and thus $V_1 = R_1/(r + \delta)$ as in equation (1). This implies $V_1(r+\delta) = R_1$; but $V_2 - V_1$ is negative and thus investors lose money over the period. Over period one in the numerical simulation of a demand shift, investors lose \$300 as in (3). Losses continue over every holding period. Investors would similarly lose money if they held static expectations after the introduction of a housing assistance program.

$$\begin{array}{ll} V_1(r+\delta)=900(.05 + .01) & R_1+(V_2-V_1) = 54 + (897-900) \quad (3) \\ = 54 & = 51 \end{array}$$

It is obvious that investors would not persist in such static expectations.

(iii) Rational Expectations

An alternative and more realistic model would assume investors value a dwelling unit with some form of rational expectations. Their expectations about future rents after a demand shift or a government housing program would recognize that rents would fall in the future as induced new construction increases the stock of housing. The formal rational expectations solution to the simulation model cannot be provided here (it is provided in endnote 3) but the intuition of the solution can be.

In order to understand market adjustment under rational expectations, consider first an increase in demand. Recall that there was an initial equilibrium, and then the demand curve shifted permanently upward at the beginning of period one and rents rose to 54. If the rent

of 54 is assumed to persist, the value of a unit would be 900 (as in static expectations). However rational expectations recognize that rents will decline from 54, and therefore the value of a dwelling in period one must be lower under rational expectations. The present value of rationally expected future rents is lower than statically expected future rents. Because under rational expectations the initial jump in unit value is lower, the induced extra construction in period one is lower. Rent in period two falls by less than with static expectations because there was less net addition to housing stock. Under rational expectations the jump in unit values is less, the immediate increase in construction is less and rents and housing stock take longer to return to equilibrium.

If investors fully understand the rental market and know all investors will act like they do, then values will be established in each period such that investors just earn normal profits; or stated alternatively, equation (4) holds for

$$V_t(r+\delta) = R_t + (V_{t+1} - V_t) \quad (4)$$

every period. This is sometimes called a perfect foresight condition. Investors set values, recognizing that the induced future new construction over many periods will yield a lower unit value next period; and the actual construction which occurs is exactly what was expected so equation (4) holds.

Under a rational expectations solution, investors earn normal returns in each period (equation (4) holds). As well, rent is

determined in the rental market where the housing stock supply curve SS intersects the demand function DD ; and new construction is determined by where the investor demand curve II intersects the construction curve CC . The basic model remains as in Figure One and Figure Two; but valuation does not follow equation (1). Rather, valuation uses rational expectations or perfect foresight. (For detailed analysis of such models see Poterba (1984), Begg (1982) or Sheffrin (1983).)

It can be shown that there exists a stable rational expectations (or perfect foresight) solution. This solution has been calculated for the numerical simulation model.³ The time path of adjustment to a demand shift is reported in Table One. As the previous intuitive argument suggested: the initial jump in values is less, construction responds more slowly, rents remain higher, and the market takes longer to reach the long-run equilibrium. After 8 periods, 22 percent of the gap between period one value and the long-run equilibrium has been closed. Similarly housing stock has made only 22 percent of the complete adjustment. Under static expectations, 28 percent of the adjustment had been made after 8 periods.

Again the rational expectations approach could be used to analyze a rental assistance program. Suppose the government offered mortgages at less than market interest rates. Investors would substitute government financing for private financing and a given stream of expected future rents would yield a higher after-tax cash flow. Static expectations would expect current rents to persist and V would rise accordingly. Rational expectations would recognize that the assistance program would bring forth more construction and therefore that rents would decline in

the future. The initial V under rational expectations will be lower than under static expectations; and the induced increase in construction will be lower in each period.

Consider a numerical example of a government mortgage lending program using the simulation model developed previously. With rational expectations the perfect foresight condition again holds - the costs of holding the stock just equal the returns - but it becomes (5) when mortgage financing is explicitly introduced. It is assumed the loan-to-value ratio is 0.75 with or without government program.

$$V_t(0.25r + 0.75r_m + \delta) = R_t + (V_{t+1} - V_t) \quad (5)$$

If $r = r_m$, equation (5) is the same as equation (4). Under the government program, investors can obtain mortgage loans at less than market rates (actual r_m is less than r). The mortgage assistance raises after-tax cash flows, even recognizing that the assistance program will induce more construction and therefore depress future rents. The full adjustment to the mortgage assistance program is set out in Table Two. Again the loans are available at an interest rate of 0.0403 compared to a market rate of 0.05; this was selected to imply the same long-run equilibrium stock and value as the demand shift.

With rational expectations, the housing program causes an increase in value to \$87,850 in period one; this is a smaller increase in value than static expectations. Period one construction rises to 38,831; a smaller increase than static expectations. Rents are unchanged in period one but gradually fall as new construction increases the housing stock.

To sum up and highlight the important aspects of rental markets for analysis of housing programs, consider again Table Two showing the time path of adjustment to a government housing program; the program might offer mortgages at attractive rates or give investors in rental real estate a tax advantage. The program will raise V_1 as shown. Period one rents will not change, but will fall from the original equilibrium as new stock is added. Value, construction and housing stock adjust as in the Table. The first point to highlight is that investors in rental real estate consider the present value of future after-tax cash flows. Housing assistance programs alter present and future after-tax cash flows and therefore the correct approach to analyzing the effect of rental assistance programs is to consider the present value of the assistance. This is set out in detail in another Appendix. If investors do not recognize that the housing program will alter future rents, they have static expectations and the housing market will adjust as in the top half of Table Two. However, static expectations are systematically wrong and systematically unprofitable. The rational expectations path is set out in the bottom half of Table Two. The initial jump in value is lower and the induced increase in construction is lower in each period under rational expectations; although the eventual long-run equilibrium is the same. When the focus of analysis is the time path of adjustment, it is important to specify how expectations of future rents are formulated.

Regardless of the expectations mechanism, Table Two highlights another fundamental point: housing markets adjust very slowly. The numerical model assumed a long-run elasticity of 3 which is higher than most researchers have found, and still less than one third of the

adjustment was completed after eight periods. The complete adjustment to a permanent housing assistance program will take many, many years. The critical parameters determining the path of adjustment are the price elasticities of the CC and DD curves.

Finally, it should be remembered that it is the net effect of a housing program which is at issue. Following Table Two, in the absence of government assistance annual new construction would be 30,000. An assistance program increases this in period one to 42,415 under static expectations or to 38,830 under rational expectations. The net effect was 12,415 new units under static expectations and 8,830 under rational expectations. But all construction receives the attractive mortgage. In this example, government subsidized 42,415 units for a net addition of 12,415 in period one or 38,830 units for a net addition of 8,830.

(iv) Extensions: Rent Control

In the model just presented, rents were determined by market forces. However in many Canadian rental markets, rents are controlled - they are set by regulation not by market forces. This will obviously change the process of adjustment from that outlined previously.

Consider a rental market in equilibrium at R_0 , S_0 , V_0 , and C_0 as in Figure Three. There is an unanticipated, permanent shift in demand to D^*D^* . Rather than letting rents rise to R^* , the government imposes rent controls and holds rent at R_0 . (Often rent control is modelled as pushing rents below an initial equilibrium, but this approach of holding rents after a demand shock to prevent adjustment is more realistic.) There is a new equilibrium. Rents remain at R_0 , stock

remains at S_0 and there is excess demand $S_0 S_c$. Unit values remain at V_0 and construction remains at C_0 which just replaces annual depreciation δS_0 . In this model, new construction continues after control because it is assumed real rents R_0 will continue under controls. This is an unrealistic assumption; more realistically controls over time would let real rents fall and so construction and stock would gradually decline. But this introduces complexities unnecessary for this stylized analysis.

The equilibrium with control has been specified. Now to understand the market dynamics, consider - as in the previous section - a (further) demand shock or the introduction of a permanent mortgage assistance program.

If there were a further demand shock to $D'D'$, excess demand would increase but there would be no other change in the equilibrium. R_0 , S , V and C would be unchanged. This assumes the regulatory rules are not changed in response to increased political pressure when excess demand increased.

If the government introduced a permanent mortgage assistance program, the after-tax cash flows on a new building would increase. Because rents are set by regulation, investors would be correct to use static expectations. They could assume R_0 would persist because any extra construction would not alter future rents. Using static expectations, investors would take the present value of the larger after-tax cash flows, and V would increase. Construction would increase. V and C would remain at these higher levels over time. Housing stock would increase until it reached the level where C just replaced

depreciation and then would stop growing. It is possible that the new equilibrium housing stock would be below the no control equilibrium stock level S_L , or that it would be equal to S_L , or if the assistance is very generous that it would be above S_L .⁴

Thus, adjustment to a permanent government housing program in a rent controlled market is very different from an uncontrolled market. Rents are unchanged rather than falling. Static and rational expectations are the same. Values of new units rise and stay there, rather than rising and declining. Existing units do not enjoy this increase in value. Construction levels rise and remain at the higher level rather than rising and declining.

Some rent control regimes exempt new construction, creating two rental markets - one in which rents are set by regulation and one in which rents are set by market forces. The rent levels in the exempt market depend crucially on how much demand spills over from the controlled market and what premium investors in new buildings demand to compensate for the risk of controls being extended into the exempt sector. If a government assistance program were offered to new construction in the exempt market, the adjustment patterns would be basically the same as those already discussed for an unregulated market and set out in Table Two.

This Section set out a model of a rental housing market. It has identified and discussed those aspects of a rental market which govern how the market responds to a permanent rental assistance program that offers mortgages at less than market interest rates. Of particular importance in understanding adjustment were the elasticity of the

construction supply curve, whether investors have static or rational expectations and whether the market is subject to rent control.

Table Three helps to summarize this analysis. It sets out how various factors affect the long-run response, measured as how they affect the percentage change in equilibrium housing stock. It also sets out how the factors affect the short-run response, measured as how they affect the percentage increase in construction in the first period.

To conclude this Section on the rental model, it is worth returning to a number of caveats mentioned at the beginning of the Section, and worth adding a few others. This stock-adjustment model had no vacancies in the rental market. With a vacancy model, adjustment in rents is somewhat slower. With a demand shift, vacancy rates fall and then rents rise, which slows the increase in construction. With a mortgage assistance program, vacancy rates rise and then rents fall. The delayed fall in rents increases the early period level of construction. The model did not explicitly analyze household formation and did not explicitly show that when rents fall the number of households in the population will tend to rise. One could presume that changes in the number of households was already captured in the demand curve. And the model did not simultaneously analyze the rental and ownership markets or explain tenure choice. If an ownership sector were allowed, when rents rise some households would leave the rental market. The impact of a demand shift is mitigated. When rents fall some households would enter the rental sector. The impact of a rental assistance program on construction and stock is increased, but rents do not decline as much. To introduce any of these enrichments is impossible graphically, would

significantly increase the complexity of the qualitative discussion and dramatically increase the complexity of formal models. Nonetheless, each is important.

Finally, the stock-adjustment model looked only at the housing market. It was partial equilibrium rather than general equilibrium. But the government housing assistance must be financed in some way. For example, if the government makes mortgage loans, it must either decrease other loans or increase its own borrowing. An increase in government borrowing may raise interest rates (or squeeze out other borrowers) and so decrease investment in the economy including the housing sector. Also, when the construction industry expands it draws resources from other sectors. Thus a housing program will decrease the output levels of other sectors. An analysis of the effects of housing programs on employment and government finances will have to take account of these general equilibrium effects. This is done in the larger project.

3. A TAXONOMY OF RENTAL HOUSING PROGRAMS

The previous section set out a model of the rental housing market which could be used to analyze the effects of government rental assistance programs. The focus of analysis was the market and the critical features of rental markets which shape how it adjusts over time. In the discussion only one simple type of assistance program was considered: the government made mortgage loans at less than market interest rates. The program was available to all investors and was permanent. However, not all government assistance programs are of this

sort; indeed governments have used an array of different programs. The four studied in this project - LD, ARP, MURB and CRSP - are quite different from one another. These differences influence not only the long-run effect of the programs on rents, unit values, housing stock and annual construction but also the time path of adjustment of these variables. This section sets out a taxonomy of rental housing programs designed to highlight those terms of an assistance program which will influence both the long-run and short-run effects of programs. The taxonomy was designed, recognizing the four specific programs studied in this project but also attempting to be general enough to deal with other programs. The taxonomy characterizes rental programs in four ways, according to (i) the terms of the assistance, (ii) the duration of the program (iii) whether the assistance is available to all investors or is limited in some way, and (iv) whether the assistance carries restrictions on the rents which can be changed.

(i) Terms of Assistance

Government could make investment in rental housing more attractive in a number of ways. It could offer grants, mortgage loans on more attractive terms than could be obtained in the private market, or it could offer reduced tax on the income from the new rental project. All of these means were utilized in the programs studied in this project. But other means are also available. The principal other means are to provide mortgage insurance at less than market rates (a loan guarantee is free insurance), to provide inputs at less than market prices (for

example, land) and to exempt or change regulations governing the project (for example, changing the zoning to permit higher density).

The possible grant structures are obvious: there could be lump sum grants, and grants over a number of years, either of which could be conditional on some aspect of the building project. For example, the grant could depend on the actual rental income and vacancy rate and be set to ensure a certain rate of return.

Loan assistance can take many forms. The loans can be at a lower interest rate, a higher loan-to-value ratio, for a longer term or a longer amortization period. The government loans could also be a type of mortgage not yet available in private mortgage markets; for example, the government might be the first to offer graduated payment mortgages or shared appreciation mortgages.

Tax advantages can also take several forms. The rate of tax on the project could be reduced, although this would be difficult because firms and individual investors pool their projects, computing tax on the whole rather than the separate parts. More commonly the allowable depreciation on a building is increased. The rules regarding capitalization versus expensing of construction period costs can be changed. For investors with other rental income, it is preferable to expense (i.e., deduct at time of expenditure) all construction period costs. And the rules regarding rental losses generated by Capital Cost Allowance (CCA) could be changed. If rental losses generated by CCA are allowed to be deducted against other income, investment in rental buildings becomes more attractive to investors wishing to shelter current income and who do not have a stream of rental income against which losses could be applied.

(ii) Duration

An important characteristic of an assistance program is whether it is permanent or temporary. So far in this Appendix, only permanent programs have been discussed. But many programs - for example MURB and CRSP - are announced as temporary assistance. The investor can retain the assistance in the future - for example the attractive mortgage - but at some time in the future investors will no longer be able to get assistance.

Permanent programs have already been analyzed. Their effects depend importantly on the elasticity of the construction curve, the nature of expectations and whether there are rent controls.

Temporary programs have different results in both the long run and short run. In a market without rent controls, a temporary program will have no long-run effect. Suppose the government made attractive mortgage loans available for one year. For one year V would rise, C would be higher and at the end of the year S would be higher. But in the next year rents would fall, and V and C would decline. New construction would not be able to replace depreciation and S would decline over time until it returned to the original equilibrium level. For a one-year temporary program, static and rational expectations would be virtually identical. Rational expectations would recognize the program ends after one year and would recognize the program will have little effect on future rents. The short-run impact of a temporary program depends upon the present value of the assistance and the elasticity of the CC curve.

A temporary program will also have no long-run effect if the private market is rent controlled. Recall Figure Three, illustrating an

equilibrium after a demand shock and the imposition of rent control. A temporary program will raise V , increase construction and the housing stock. (The V increase under rational or static expectations is the same because rents are set by regulation.) But this increase in S is not a permanent increase, because after the program is terminated construction will return to its old level, but will not replace depreciation on the larger housing stock. Over time, the housing stock shrinks until it returns to its old level, and construction replaces depreciation.

(iii) Availability

All the rental programs discussed to this point have been available to all investors; but there can be limits on availability and these limits affect the long-run and short-run effects of programs. Tax programs are usually available to all investors, although there may be limitations on the type of building eligible for the tax assistance. Very often, rental programs are available to a limited number of investors because the government has budgeted a limited amount for assistance over the year. There can also be restrictions on the geographic availability of funds, for example, assistance may only be available for buildings in certain provinces, certain regions or to housing markets with low vacancy rates. There can be restrictions on the type of building eligible for assistance, most commonly there are limits on the value of units. By restricting assistance to buildings with lower rents, it is hoped "deserving" tenants will enjoy some of the benefits.

For this discussion, the two most common limitations on availability will be considered: limits due to a government budget constraint and limits on the value of a unit eligible for assistance.

Suppose that annual construction in equilibrium is C_0 and then the government initiates a rental assistance program which raises the value of a unit from V_0 to V_1 under static expectations, but only B units can be assisted and B is less than C_0 . Such a program will have no short-run or long-run effects, as in Figure Four. The first B units built receive the assistance, but they would have been built anyway. For units built after B , the value is the same as it was in the initial equilibrium because no assistance is available. The II curve becomes $I'I'B'I$ in Figure Four. The equilibrium level of construction remains at C_0 . (The results are the same under either static or rational expectations.) The analysis of Figure Four assumes that all building projects are the same and that their costs vary as the level of total construction varies and causes input prices to change. However, it is possible that building projects, with given input costs, are of different profitability. Construction occurs until the marginal project is built. If the government could identify the marginal projects it could use a limited budget to increase total construction - indeed it could ensure that all assistance would flow to projects which otherwise would not have been undertaken. But these are the least efficient or least desirable or least profitable projects. (The analogy to a limited budget for regional development is exact. The government can subsidize a firm to build an industrial plant in a location where it otherwise would

not, but this is an inefficient or unprofitable plant which would not exist without the assistance.)

Assistance which is limited to units of below a certain value is rather difficult to model. This complexity cannot be introduced into the stock-adjustment model, strictly speaking, because all units are assumed to be the same and to command the same price. If the analysis were done in terms of housing stock, and dwelling units could contain different quantities of housing stock (see endnote 1), restrictions on unit values could be introduced but the result would be the same as Figure Four: the program would have no short-run or long-run effect on construction or housing stock. If the B assistance was for more small units than had been constructed in equilibrium, then more of these types of units would be built, but the total quantity of housing stock constructed would be unchanged.

To properly introduce unit value limits into the analysis requires a model in which dwelling units vary in quality or in their characteristics. Rental units are heterogeneous, and the prices of different types of units are different. The initial equilibrium depends on the preferences of households and the technologies of firms and this equilibrium specifies the types of units which are produced and their prices. Then a government program could be analyzed which assists one type of unit. The program would change the output levels and prices of many types of units, not just the type assisted. See Sweeney (1974) for an example of this type of analysis.

(iv) Rent Restrictions

The final characteristic of the assistance is whether there are restrictions on the rents which can be charged. Rent restrictions ensure that some of the value of the assistance flows to tenants in the form of reduced rents rather than entirely to the investor as an increased rate of return. Sometimes further restrictions are added which specify what sorts of tenants can live in the building. Such restrictions attempt to limit those who live in the building to "deserving" tenants. The tenant restrictions presumably either reduce a landlord's flexibility or force the landlord to take less desirable tenants and therefore reduce the value of the assistance to the investor.

Rent restrictions, in effect, set up a separate rental market outside the private market in which rents are set by the terms of assistance rather than by market forces. In order to analyze the effects of such a program consider an initial equilibrium, as in Figure One, of an uncontrolled rental market; and the government introduces a mortgage assistance program with restrictions on the rents which can be charged. It is assumed that the combination of assistance and reduced rents still leaves an investor better off than with an unassisted project. Therefore, investors prefer the assisted projects to the unassisted. Usually with such programs, the government sets a limit on the number of units it will assist in one year. (If this were not true, there would be no construction in the unassisted private market.) Investors (or non-profit groups) will initiate new buildings until the government's limit is reached. The number of assisted units constructed in any period is thus set by the government budget constraint (assuming assisted projects

are more profitable than unassisted projects); and rents in these buildings are set by regulation, not market forces.

Although such government programs establish a regulated rental market separate from the private market, they do influence the private market. The influence moves through two channels. First, households move from the private rental market to the assisted market, which shifts the private rental demand curve down and left. The extent of the shift depends on how many units the government has assisted - i.e. how many households can be accommodated in the assisted sector - and on how household formation is affected by the programs. The simplest pattern would have one household move from the private to the assisted market. Often, however, those taking the assisted units previously lived in crowded conditions; the old crowded household splits with some members remaining in private market and some members moving to the assisted market. With such cases, the downward shift of the private market demand curve is mitigated. The second channel of influence is through construction costs. The building of assisted units expands the output of the construction industry and raises input prices - the prices of land, labour and materials. The private CC curve shifts up and left.

Thus the government program causes the private demand curve to fall and so causes private rents to fall; the program also causes the construction supply curve to rise and so causes private construction to fall. Private construction falls still further because private rents fall. In the long-run, private construction declines will considerably offset the new construction financed under the assistance program. The total housing stock will be only slightly larger.

The exact time path of net new construction is hard to predict because of the uncertainties about household formation. Again the time path would be influenced by whether private investors had static or rational expectations; and the time path and adjustment would be influenced by whether or not the private market were rent controlled.

This project analyses four rental housing programs, of quite different sorts. Tables Four to Seven characterize these programs according to the taxonomy of this section.

This Section has emphasized how characteristics in the design of housing assistance programs influence the time path and long-run response in a rental market. Of particular interest were the terms of assistance, whether the program was permanent or temporary, whether the budget for assistance was limited, and whether there were rent restrictions. Table Eight helps to summarize the analysis. It sets out how the four factors affect the long-run market response, measured as the percentage change in the equilibrium housing stock. Also, it sets out how the factors affect the short-run response, measured as the percentage change in period one construction.

Table One

Time Path of Housing Market Variables*
Response to Demand Shift

<u>Static</u> <u>Expectations</u>	<u>Initial</u> <u>Equilibrium</u>				<u>Period</u>								<u>Final</u> <u>Equilibrium</u>
		1	2	3	4	5	6	7	8	9			
Rent (\$00)	48	54	53.82	53.65	53.48	53.32	53.17	53.02	52.88	52.75	...		49.5
Value (\$00)	800	900	897	894	891	889	886	883	881	879	...		825
Construction (0,000)	3.0	4.13	4.09	4.06	4.03	4.00	3.97	3.94	3.92	3.89	...		3.28
Stock (0,000)	300	300	301.13	302.21	303.24	304.24	305.19	306.11	306.99	307.84	...		328.13
<u>Rational Expectations</u>													
Rent (\$00)	48	54	53.87	53.73	53.61	53.48	53.36	53.25	53.14	53.03	...		49.5
Value (\$00)	800	875	873.5	872.05	870.63	869.26	867.94	866.65	865.40	864.18	...		825
Construction (0,000)	3.0	3.84	3.83	3.81	3.79	3.78	3.76	3.75	3.74	3.72	...		3.28
Stock (0,000)	300	300	300.84	301.66	302.46	303.23	303.97	304.70	305.40	306.08	...		328.13

*The underlying simulation model is set out in the Endnotes 2 and 3.

Table Two

Time Path of Housing Market Variables*
Response to Mortgage Assistance

<u>Static</u> <u>Expectations</u>	<u>Initial</u> <u>Equilibrium</u>				<u>Period</u>						<u>Final</u> <u>Equilibrium</u>
		1	2	3	4	5	6	7	8		
Rent (\$00)	48	48	47.8	47.6	47.4	47.3	47.1	46.9	46.8	...	43.5
Value (\$00)	800	910	907	903	899	896	893	890	887	...	825
Construction (0,000)	3.0	4.24	4.20	4.16	4.12	4.08	4.05	4.01	3.98	...	3.28
Stock (0,000)	300	300	301.2	302.4	303.6	304.6	305.7	306.7	307.6	...	328.1
<u>Rational Expectations</u>											
Rent (\$00)	48	48	47.9	47.7	47.6	47.5	47.3	47.2	47.1	...	43.5
Value (\$00)	800	879	877	875	874	872	871	869	868	...	825
Construction (0,000)	3.0	3.88	3.86	3.85	3.83	3.81	3.79	3.78	3.76	...	3.28
Stock (0,000)	300	300	300.9	301.7	302.6	303.4	304.1	304.9	305.6	...	328.1

*The underlying simulation model is set out in the Endnotes 2 and 3.

Table ThreeResponse to a Permanent Rental Assistance ProgramLong Run

Percentage increase in equilibrium housing stock

- greater the more elastic the CC curve
- greater the more elastic the DD curve
- the same under static and rational expectations
- greater in a rent controlled market.

Short Run

Percentage increase in construction in period one

- greater the more elastic the CC curve
- not influenced by elasticity of the DD curve, with static expectations
- greater under static than rational expectations
- the same under rent controls as the static expectation of an uncontrolled market

Table FourLimited Dividend Program(pre-1968)¹Terms of Assistance

- mortgage loan
 - high ratio loan (90-95%)
 - long amortization period (50 years)
 - low interest rate (2% below market rate)

Duration

- permanent

Availability

- government budget constraint
- unit price restrictions

Rent Restrictions

- return on capital limited to 5 percent².

-
1. Post-1968, investors can pre-pay loan and remove rent restrictions. Since 1985, all projects have phase-out option.
 2. Post-1968, return on equity negotiable.

Table Five

(a)

Assisted Rental Program

(1975)

Terms of Assistance

- grants
 - non-taxable, up to \$75 per unit per month, to permit stated rate of return
 - subsidy level reduced by equal amounts over 10 years

Duration

- temporary

Availability

- government budget constraint
- unit price restrictions (?)

Rent Restrictions

- increases limited to grant reduction plus increase in operating costs

Table Five

(b)

Assisted Rental Program

(1976)

Terms of Assistance

- mortgage loan
 - annual second mortgage advance up to \$100 per unit
 - advances reduced by equal amounts over 10 years
 - interest free over 10 years; payments begin after 10 years

(Private first mortgage plus ARP second mortgage approximates a subsidized graduated payment mortgage.)

Duration

- temporary

Availability

- government budget constraint
- unit price restrictions (?)
- buildings of more than 8 units

Rent Restrictions

- set in first year, then set by market; advances of second mortgage may be altered to control return on equity

Table Five

(c)

Assisted Rental Program

(1978)

Terms of Assistance

- mortgage loan
 - annual second mortgage advances equal to lesser of:
amount needed to provide 5 per cent return on equity
versus \$225 per month per \$1,000 of first mortgage.
 - advances reduced over 10 years.
 - interest accruing on advances over 10 years; payments
begin after 10 years.

(Private first mortgage plus ARP second mortgage approximates a
unsubsidized graduated payment mortgage.)

Duration

- temporary

Availability

- government budget constraint

Rent Restrictions

- set in first year then set by market, but advances may
alter to control return on equity.

Table SixMultiple Unit Residential Buildings¹ ProgramTerms of Assistance

- tax
 - CCA allowed to create rental loss.
 - expensing of soft costs (against other income)

Duration

- temporary.

Availability

- available to all investors

Rent Restrictions

- none

-
1. A multiple unit building was defined as having at least two residential units and 80 per cent of floor space for residential use. Motels and hotels were excluded.

Table SevenCanada Rental Supply PlanTerms of Assistance

- mortgage loan
 - interest-free second mortgage, 15 year term
 - (first+second mortgage) \leq 0.8 project cost
 - maximum loan of \$7,500 per unit

Duration

- temporary

Availability

- government budget constraint
- no pooling with other assistance.

Rent Restrictions

- none

Table EightResponse of a Rental Market to Types of
Housing Assistance ProgramLong Run

Percentage increase in equilibrium housing stock

- greater the more generous the assistance
- no increase for a temporary program
- no increase if budget for assistance is limited to less than original construction level
- with rent restrictions a temporary program can have a small effect; the effect is larger if the market is rent controlled

Short Run

Percentage increase in construction in period one

- greater the more generous the assistance
- greater for a temporary program than a rational expectations permanent program
- no increase if budget for assistance is limited to less than original construction level
- with rent restrictions the increase is determined by the government budget constraint

ENDNOTES

1. In this representation, demand is measured in dwelling units. There is the implicit assumption that all households occupy one dwelling unit and that all dwelling units are identical. The market sets the rent R on this "standard dwelling unit". Obviously all dwelling units are not the same. An alternative modelling approach is to assume an unobservable theoretical commodity called housing services is exchanged in the rental market. All units of housing service are identical and the rental market determines their price. Different dwelling units provide different quantities of housing service (see Olsen 1969). Demand is measured in dwelling units in this Appendix because the econometric work of the larger project measures construction output in dwelling units.

2. The initial market demand function is:

$$R_t = 96 - .16 S_t$$

The initial housing stock is:

$$S_0 = 300$$

Housing stock changes according to:

$$S_t = S_{t-1} + C_{t-1} - \delta S_{t-1}$$

The construction supply curve is:

$$C_t = -6 + .01125 V_t$$

The initial equilibrium is:

$$R_0 = 48 \quad V_0 = \frac{R_0}{r+\delta} = \frac{48}{.05+.01} = 800$$

$$S_0 = 300 \quad C_0 = 3$$

This equilibrium can be thought of as rent of \$4800 per year (\$400 per month), a dwelling unit value of \$80,000, a housing stock of 3,000,000 units and annual construction of 30,000. These are reasonable values for the Canadian rental in the 1980s.

The shifted demand function is:

$$R_t = 102 - .16 S_t$$

3. Let \hat{V} be the change in V per time period and \hat{S} be the change in housing stock. The adjustment of the rental market is described by the differential equation system:

$$\begin{aligned}\hat{V} &= (r+\delta) V - R \\ &= .06V + .16S - 102\end{aligned}$$

$$\begin{aligned}\hat{S} &= .01125V - \delta S - 6 \\ &= .01125V - .01S - 6\end{aligned}$$

The phase diagram for this equation system is Figure Five.

The equilibrium (S^*, V^*) is a saddle point. The property of a saddlepoint is that there is a unique convergent path PP to (S^*, V^*) , a path which never crosses the $V^*=0$ or $\hat{S}=0$ lines. The rational expectations solution is this convergent path along which:

$$\begin{aligned}\hat{V} &= \lambda (V - V^*) \\ \hat{S} &= \lambda (S - S^*)\end{aligned}$$

where λ is the negative eigenvalue of the coefficient matrix of the differential equation system. For this model $\lambda = -.03$. The equation of the convergent path is:

$$V = 1408.3 - 1.6S$$

This housing model is due to Poterba (1984). It is clearly discussed in Sheffrin (1983). The solution technique is discussed in Begg (1982).

The original equilibrium is E_0 . The $\hat{V} = 0$ line shifts up as the demand function changes, as in Figure Six.

Under static expectations V jumps to M on the new $\hat{V} = 0$ line and moves down this line to the new long-run equilibrium, E_1 . Under rational expectations, V jumps to R and moves down the convergent path PP to E_1 .

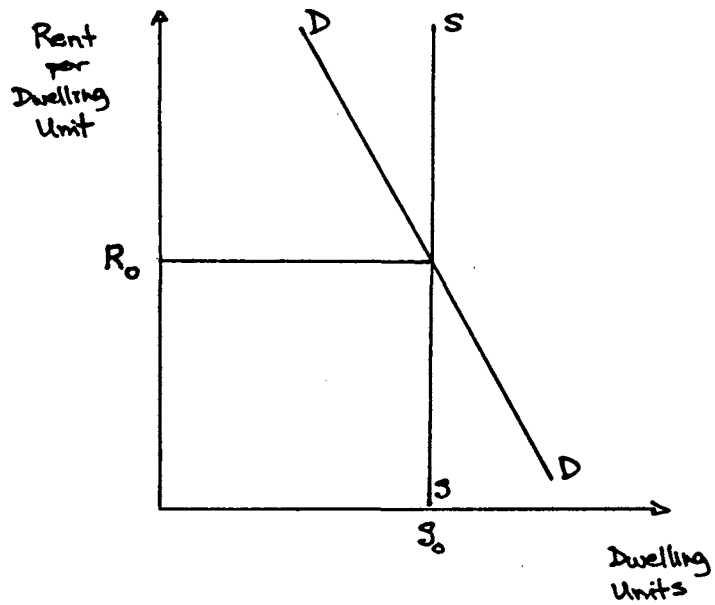
4. The mortgage assistance in this model is a perpetual loan at less than market rates. If the assistance is so generous as to increase the stock above S_L , controls on rents would not be binding; and investors would have to recognize that new construction would reduce future rents.

REFERENCES

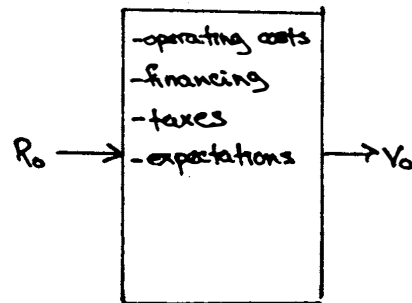
- Begg, D. (1982) The Rational Expectations Revolution in Macroeconomics (London: George, Allen, Unwin).
- Fallis, G. (1985) Housing Economics (Toronto: Butterworths).
- Olsen, E.O. (1969) "A Competitive Theory of the Housing Market", American Economic Review 59, 612-622.
- (1986) "The Demand and Supply of Housing Services: A Critical Survey of the Empirical Literature", in E.S. Mills, ed., Handbook of Urban Economics (Amsterdam: North Holland).
- Poterba, J.M. (1984) "Tax Subsidies to Owner-Occupied Housing: An Asset Market Approach", The Quarterly Journal of Economics, 729-752.
- Sheffrin, S.M. (1983) Rational Expectations (Cambridge: Cambridge University Press).
- Smith, L.B. (1974) The Postwar Canadian Housing and Residential Mortgage Market and the Role of Government (Toronto: University of Toronto Press).
- (1987) Adjustment Mechanisms in Real Estate Markets (New York: Salomon Brothers Inc.).
- Sweeney, J.L. (1974) "A Commodity Hierarchy Model of the Rental Housing Market", Journal of Urban Economics, 1, 288-323.
- Topel, R. and S. Rosen (1988) "Housing Investment in the United States", The Journal of Political Economy, 96, 718-740.

FIGURE ONE

RENTAL
MARKET



DETERMINATION
OF
DWELLING UNIT
PRICE



CONSTRUCTION
MARKET

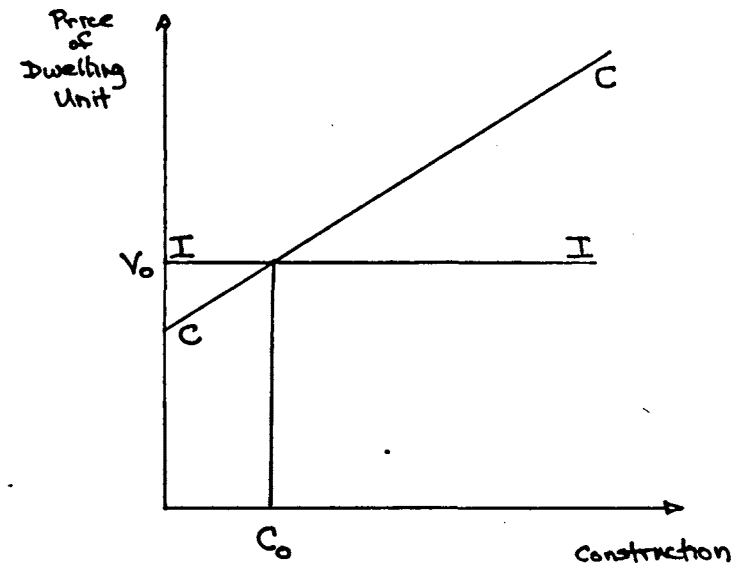
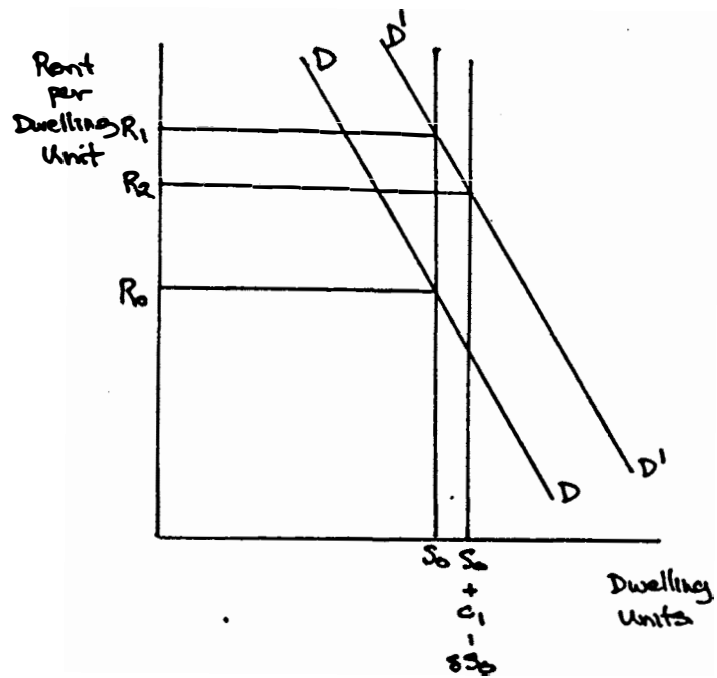
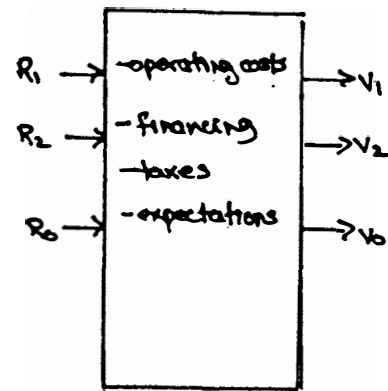


FIGURE TWO

RENTAL MARKET



DETERMINATION OF DWELLING UNIT PRICE



CONSTRUCTION MARKET

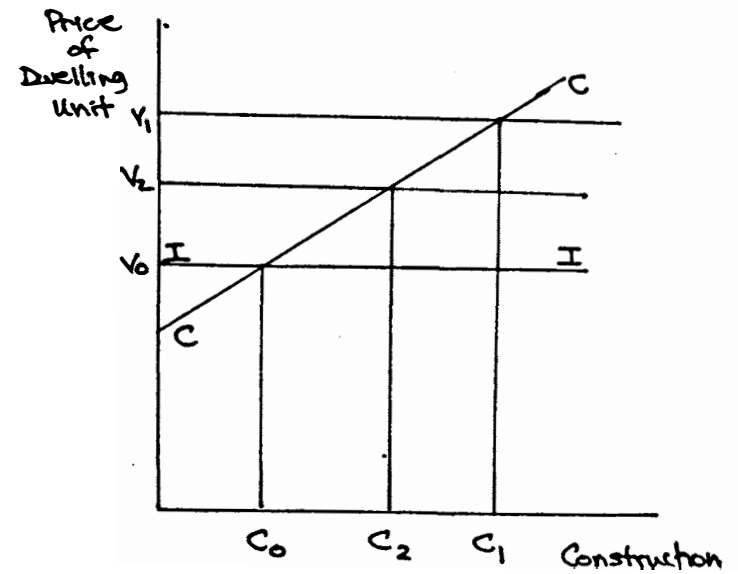


FIGURE THREE

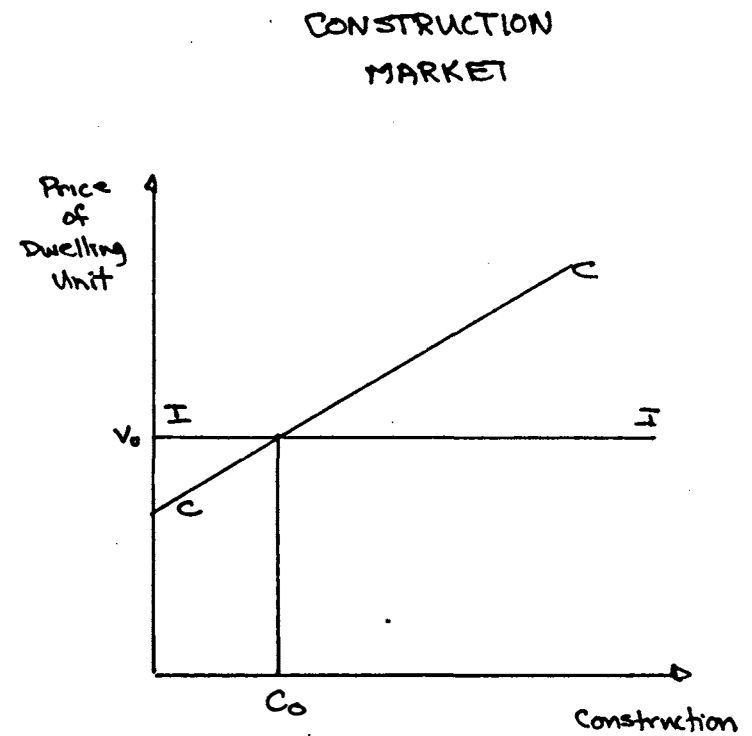
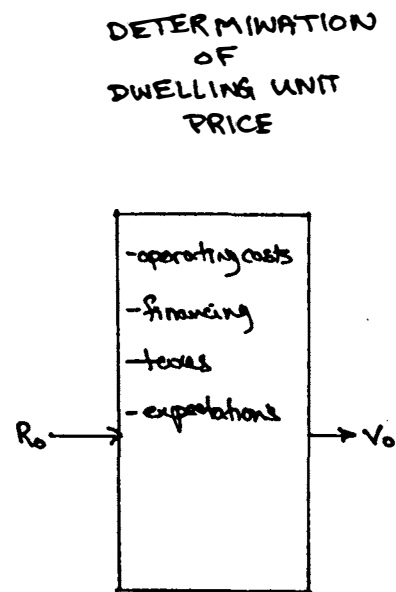
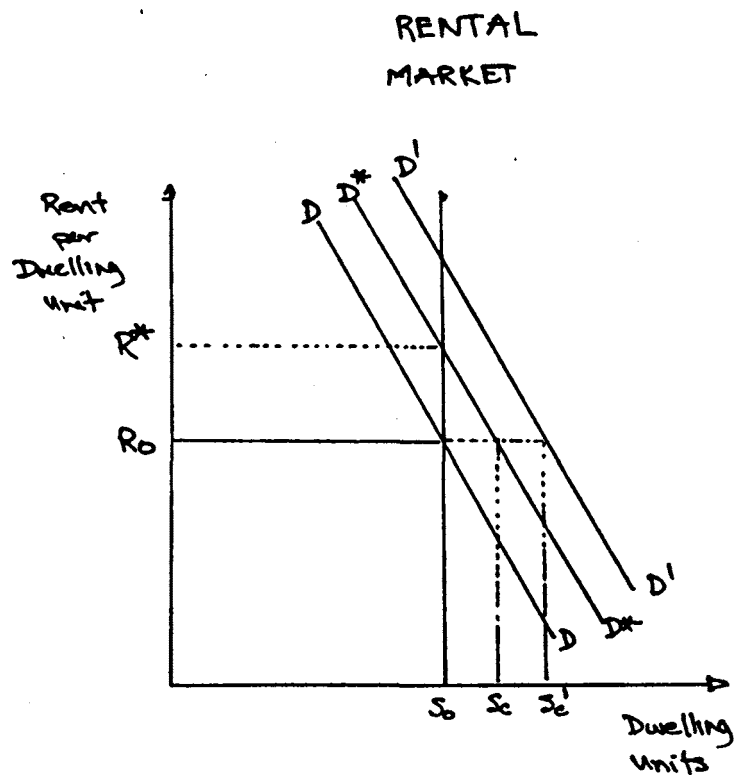


FIGURE FOUR

CONSTRUCTION
MARKET

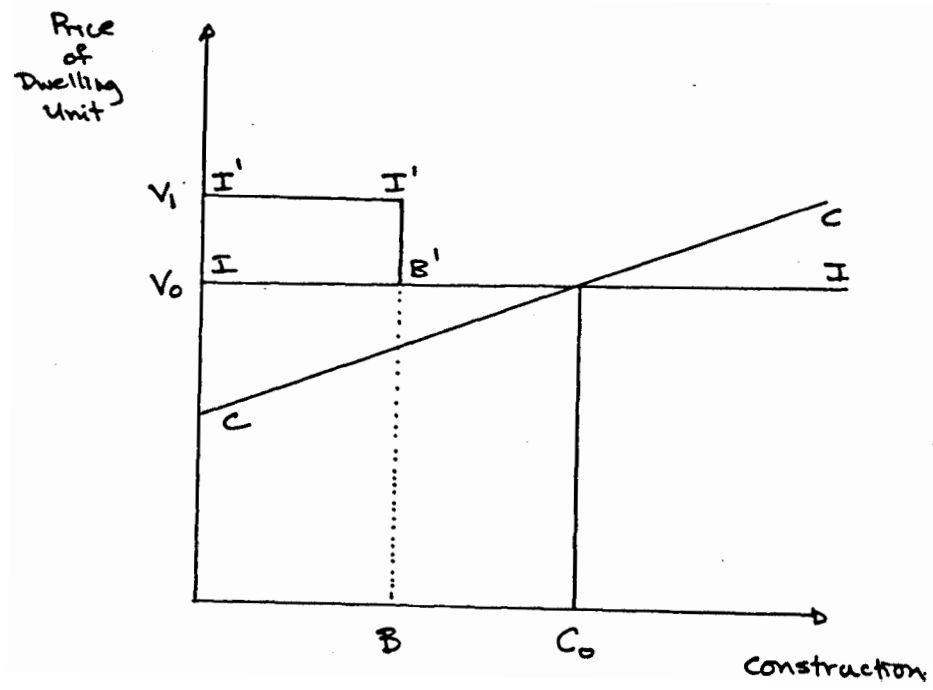


FIGURE FIVE

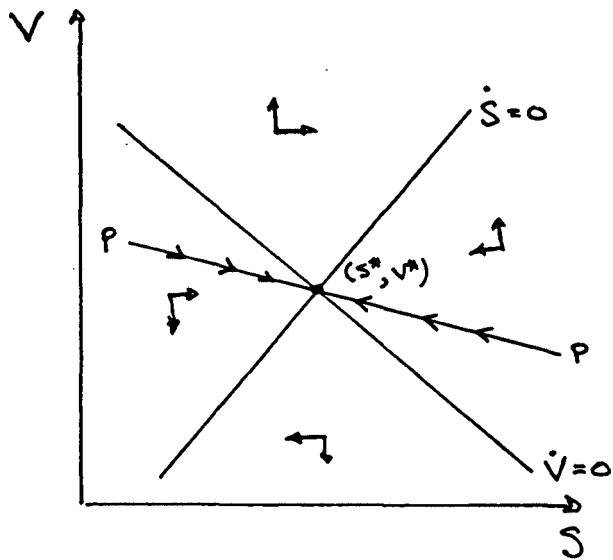
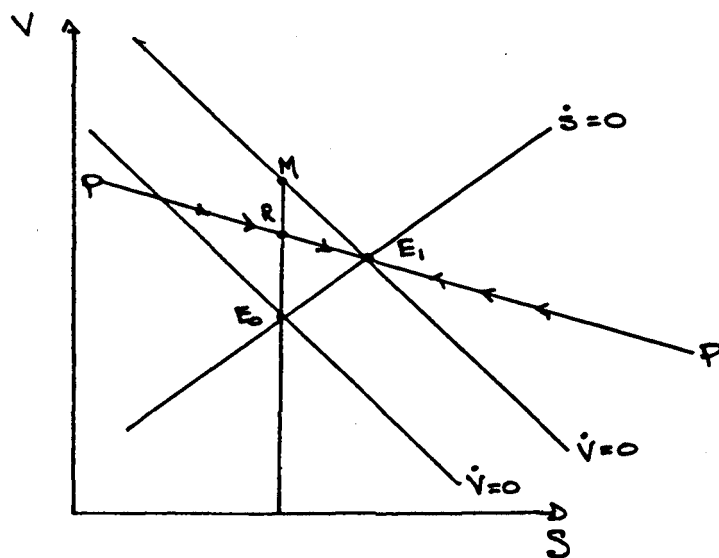


FIGURE SIX



May 16, 1989

Appendix C

AN ECONOMETRIC MODEL OF THE
RESIDENTIAL HOUSING MARKET
IN CANADA

Institute for Policy Analysis

University of Toronto

An Econometric Model of the Canadian Residential Housing Market

1. Introduction/Overview

This Appendix describes attempts to formulate and empirically estimate the parameters of an econometric model of the residential housing market in Canada. The model is formulated on a basis consistent with received economic theory and most of the model's behavioural equations may be deduced from optimizing behaviour by economic agents involved, variously, in the consumption of housing services or the production of housing facilities.

The model's parameters are estimated using quarterly data from sample periods embraced by the time interval from 1961:1 through 1988:3. The sample period used in the estimation of the parameters differs from equation to equation, depending upon the availability of time series data for the variables that appear in the equation.

As represented here, the residential housing market is decomposed into two inter-related sub-markets. The first describes the determinants of supply and demand and the price-equilibration mechanism operative in the economy-wide sub-market for single-detached houses. The second depicts the analogous features of the economy-wide sub-market for multiple-unit residential structures.

In actuality the residential housing sector in Canada is comprised of a large number of local markets for both owner-occupied housing and rental housing. An ideal representation would identify demands and supplies of owner-occupied and rental units in each locality throughout

Canada. The two kinds of housing are linked locally by cross substitution possibilities in demands and supplies (including the possibility of converting rental into owner-occupied units and vice versa). The local sub-markets are linked via inter-locality migration and an economy-wide market for financial capital.

The main obstacle to developing a model of local sub-markets is a lack of high-quality data pertaining to these sub-markets. Data that identify prices, quantities, and qualities of owner-occupied stocks and stocks of rental housing at the local level are non-existent. Indeed, these data do not even exist at the national level.

The representation presented here is a compromise based on data availability. We model the housing market on an economy-wide basis.

The single-detached sub-market proxies for the owner-occupied sub-market at the aggregate level. That is, we view the demand for single-detached units as arising from the factors that motivate owner-occupants.

Causal observation suggests that the large majority of single-detached units are owner-occupied. Further, there is no theoretical basis for believing that the factors that cause owner-occupants to purchase (or not purchase) would differ from the factors that motivate owner-leasers. Both types of agents should view home ownership as a form of investment and be profit-motivated. The main factor that differentiates these two types of agents is a difference in the tax treatment of the "returns" from this type of investment. This feature of the Canadian income tax system is ignored in what follows, and all single-detached units are modelled as if they are owner-occupied.

The multiple unit sub-market is used to represent the rental portion

of the national housing market. That is, the model is structured as if all occupied multiple units are occupied by rent-paying tenants. This is clearly not an accurate depiction of the true state of affairs. Multiple housing units consist of semi-detached and row-houses as well as units in apartment building and condominiums. Significant fractions of semi-detached houses, row-houses and condominium units must surely be owner-occupied, but precise data on these fractions and their price and quality characteristics are not available.

One feature that distinguishes the owner-occupied and rental sub-markets has to do with price-equilibration dynamics. Owner-occupied dwellings generate a flow of housing services valued by the owner-occupant in the manner of a shadow price. This shadow price is not constrained by the artificial barriers to price adjustment that exist for some market prices (e.g., forward contracts, rent controls) and can be viewed as adjusting instantaneously to changes in market conditions. For example, an event (such as a rise in incomes) that increase the demand for the housing services of owner-occupied units should cause an immediate rise in the shadow price of those services. This, in turn, has implications for the behaviour of the market price of owner-occupied units.

In principle, the market price of any housing unit will be equal to the capitalized value (or present discounted value) of the net after-tax flow of current and expected future rents associated with the property. In the case of an owner-occupied unit, the market price will be equal to the capitalized value of the shadow price of its current and expected future flow of housing services. Since the shadow price adjusts

instantaneously to market events, so too will the market price of the unit. In other words, the market price of an owner-occupied unit may be expected to always be such that it equates demand with supply for the unit.

In our representation of this an index of the prices of new single-detached housing units is modelled as equating the stock demand for single units with the stock supply in each time period. The single-detached market always clears.

In contrast, rents charged on rental housing units are usually determined on the basis of forward contracts. The rent charged during any one-year period is typically set at the beginning of the period and not subject to change over the duration of the contract. With this kind of arrangement rents are not free to equilibrate the demand and supply of rental housing in the short-term. Once rents have been set, disturbances to demand and supply in the rental market are accommodated in the short-term by movements in occupancy rates.

Longer-term movements in rents can be expected to move in the direction of equilibration as existing contracts expire and new contracts are negotiated. A current-period rise (fall) in occupancy rates will signal a rise (fall) in the future values of rents. The direction of movements in future rents will be in the direction of eliminating the current-period imbalances between demand and supply.

The existence of government-imposed rent controls may interfere with the long-term equilibration process. Binding limits on upward movements in rents in the face of excess demand for rental accommodation will surely slow the long-term adjustment of rents and may, in certain

circumstances, prevent the equilibration process from being completed. The latter would lead to a situation of permanent queuing by would-be renters who are unable to secure accommodation.

The slow response of rents to market conditions has implications for the selling prices of rental units.

A permanent increase in the demand for rental accommodation will raise the market prices of rental buildings by causing the capitalized value of expected future rents to rise. Market prices will be expected to rise in the future as the time period approaches the date at which future rents are expected to rise. The existence of binding rent controls will reduce the responsiveness of market prices to current market conditions. But even in the absence of rent controls, the rental sub-market is characterized by slow and partial adjustment of rents, selling prices of buildings, and quantities.

We model the sub-market for multiple units as strictly a rental sub-market. The stock demand is described by a partial adjustment equation. Rents are represented as responding to vacancy rates with long lags. The length of the lags and the nature of the response mechanism is permitted to differ before and after 1974, when rent controls were introduced in British Columbia and, a year later, in most provinces at the urging of the federal government.

For both single-detached and multiple housing units the stock supply at the end of any time period may be computed as the stock supply at the end of the preceding time period, plus the period's completions of newly constructed units, less net removals from the existing stock. Net removals are modelled as a constant fraction of existing stock for each

kind of housing. Completions are modelled as a distributed lag of recent past housing starts for each sub-market. The key behavioural equations in the supply sides of the two sub-markets are equations determining housing starts. Producers of new housing units are modelled as price-taking profit maximizers facing rising marginal cost schedules in both the short and the long run. The marginal cost of producing single-detached (multiple) units is increasing in the periods' total volume of single (multiple) starts. The model also allows for the possibility that producers may face costs of adjusting the level of production from one time period to the next.

Profit-maximizing behaviour equates the marginal cost of starting a particular type of housing unit with its market (selling) price. Thus, singles starts are modelled as an increasing function of the market price of single-detached houses. Similarly, multiple starts are increasing in some measure of the selling price of multiple units.

Some multiple-unit starts were eligible for subsidies under various CMHC programs that operated over different parts of the sample period. To capture these effects and obtain estimates of the net impacts of the subsidy programs on multiple starts, constructed variables that measure the contribution of each program to profits from owning a prototypical apartment building are included as additional independent variables in the equation determining multiple-unit starts. A complete description of these constructed variables appears in Appendix A.

Designing the multiple-unit sub-model in a manner that permits the net impacts of the CMHC subsidy programs to be identified, while at the same time addressing the complications associated with rent controls,

proved to be a most challenging aspect of the study. The effects of rent controls that do not operate uniformly across the country are difficult to quantify.

The remainder of this Appendix is organized as follows: Details of the model's equations are described in Sections 2 and 3. Section 2 describes the sub-market for single-detached units, while Section 3 describes the sub-market for multiple units. Empirical results are presented in Section 4 for the single-detached sub-market and in Section 5 for the multiple-unit sub-market. A listing of variable definitions and data sources is presented in Section 6.

2. The Sub-Market for Single-Detached Units

Demands for housing services are described by a standard linear, or log-linear, consumer demand system. A representative consumer is assumed to have preferences defined over three "goods": real housing services provided by single-detached houses, real housing services provided by multiple housing units, and all other consumer goods and services, collectively. The "goods" are taken to be gross substitutes on a pair-wise basis. The representative consumer chooses among the three "goods" in a manner that maximizes a standard utility function, subject to the usual budget constraint.

The real housing services derived from single-detached housing units in any time period are assumed to be strictly proportional to the end-of-period stock of single-detached houses. This assumption has been adopted in virtually all previous studies of the residential housing market in Canada and the U.S. (e.g., Kearle [1979], Poterba [1984], Topel and Rosen

[1988], and Smith [1969]).

The foregoing assumptions yield a stock demand equation for single-detached units in per capita terms that is expressed as

$$(1) \quad \ln(K_{S_t}^*/N_t) = b_0 + b_1 \ln(Y_t/N_t) + b_2 \ln(S_t/CPI_t) + b_3 \ln(R_t/CPI_t) \\ + \epsilon_{S_t},$$

where

- $K_{S_t}^*$ - the desired stock of single-detached units at the end of time period t .
 - N_t - population during time period t .
 - Y_t - real personal disposable income during time period t .
 - S_t - the service price, or user cost, of single-detached houses during time period t .
 - CPI_t - the all-items consumer price index during time period t .
 - R_t - an index of rents paid for rental accommodation during time period t .
 - ϵ_{S_t} - a stochastic disturbance term in time period t .
- b_0, b_1, b_2, b_3 are parameters.

An essential property of Equation (1) is that it is homogeneous of degree zero in nominal disposable income and all prices. This property is imposed by economic theory, but theory does not dictate the precise form of the equation. Equation (1) is written here as linear in the natural logarithms of the variables. An equation that is linear in the levels of the variables would be equally acceptable from a theoretical

viewpoint. The choice between logarithms and levels is an empirical issue and some experimentation with these alternative forms is presented in the section dealing with empirical results. For the most part logarithmic versions of equations performed slightly better in the empirical tests, and so the entire model is described here using logarithmic specifications.

The gross-substitutes assumption noted above imposes the following restriction on the parameters:

$$b_1 \geq 0, \quad b_2 \leq 0, \quad b_3 \geq 0 .$$

The actual end-of-period stock demand ($K_{S_t}^D$) is represented as a partial adjustment mechanism. Specifically:

$$(2) \quad \ln(K_{S_t}^D / N_t) = \gamma_S \ln(K_{S_t}^* / N_t) + (1-\gamma_S) \ln(K_{S_{t-1}} / N_{t-1}) ,$$

where $K_{S_{t-1}}$ denotes the actual stock of single-detached units at the end of time period $t-1$ and γ_S ($0 < \gamma_S \leq 1$) is a speed of adjustment parameter.

As discussed in the Introduction, prices ought to be fully flexible in a market in which all occupants are also owners. We would expect this flexibility to extend to quantity demands as well, so γ_S should be close to 1.0 if all single-detached units were owner-occupied. However, the inclusion in the singles sub-market of some rental accommodation prompted us to at least allow for the possibility that there may be some partial adjustment in stock demands in this sub-market.

In estimating equations for the singles sub-market we permit γ_S to

be a free parameter in some regressions and constrain $\gamma_S = 1.0$ in others. As it turns out, the preferred estimated equation has $\gamma_S = 1.0$. Nonetheless, the remainder of the description of this sub-market proceeds with the more general specification associated with Equation (2).

The price of single-detached units, P_{S_t} , is a sub-component of the user cost variable, S_t , which appears in the R.H.S. of Equation (1). (Details on this are to be presented shortly.) It is convenient for now to regard S_t as the "price" that equilibrates the stock demand and stock supply in each time period t . Setting $K_{S_t}^D$ equal to K_{S_t} and solving Equations (1) and (2) for the equilibrium value of the user cost yields.

$$(3) \quad \ln(S_t/CPI_t) = -\frac{b_0}{b_2} - \frac{b_1}{b_2} \ln(Y_t/N_t) - \frac{b_3}{b_2} \ln(R_t/CPI_t) \\ + \frac{1}{\gamma_S b_2} \ln(K_{S_t}/N_t) - \frac{(1-\gamma_S)}{\gamma_S b_2} \ln(K_{S_{t-1}}/N_{t-1}) - \frac{\epsilon_{S_t}}{b_2}$$

Equation (3) is an estimable equation from which all of the parameters are recoverable. The parameters are also recoverable from regressions of a re-normalized variant of this equation that has $\ln(K_{S_t}/N_t)$ as the dependent variable and $\ln(S_t/CPI_t)$ as one of the regressors (along with the other variables on the RHS of Equation (3)).

From an econometrics viewpoint, Equation (3) is the more appropriate normalization for estimation purposes. The random disturbance, ϵ_{S_t} , is bound to be correlated with $\ln(S_t/CPI_t)$ but can

be expected to have virtually no contemporaneous correlation with any of the other variables appearing in the equation.

ϵ_{S_t} is a random disturbance to the stock demand during time period t . Since K_{S_t} is effectively a predetermined variable (the period t value depends on $K_{S_{t-1}}$ and lagged values of single-detached housing starts), it must be contemporaneously uncorrelated with ϵ_{S_t} . The contractual nature of rents assures that the contemporaneous correlation between ϵ_{S_t} and R_t will be zero or small, and Y_t is exogenous to the housing sector. The effects of surprise realization in ϵ_{S_t} are absorbed by movements in the contemporaneous user cost, S_t , making the latter the appropriate dependent variable in a regression to estimate γ_s and b_j ($j = 0, 1, 2, 3$).

The User Cost and the Price of Single-Detached Units

An expression relating the user cost to the market price of single-detached units, denoted here by P_{S_t} , is readily derived.

Let w_t denote the nominal shadow price associated with the imputed housing services of a marginal unit of owner-occupied housing stock per capita during time period t . It is a straight-forward exercise to show that $\ln(w_t)$ is equal to $\ln(CPI_t)$ plus the R.H.S. of Equation (3). Observe that this implies that w_t is a decreasing function of (K_{S_t}/N_t) ; i.e.,

$$\frac{\partial w_t}{\partial (K_{S_t}/N_t)} < 0.$$

What is going on here is that the representative consumer has

preferences defined over three "goods". The shadow price of single-detached housing services is associated with the marginal utility derived from consuming this "good". This marginal utility is decreasing in the consumption of this "good" and non-decreasing in the consumption of the other goods. The prudent consumer will acquire single-detached housing up to the point where the shadow price of the marginal unit of (K_{S_t}/N_t) is just equal to its nominal cost, S_t , which implies Equation (3). Utility maximization equates w_t with S_t .

Another way of stating the same thing is to say that the equilibrium expected rate of return on investment in a marginal unit of single-detached housing will be equated with the risk-adjusted expected rate of return on the next best alternative investment. Formally, the equilibrium condition is

$$(4) \quad \frac{w_t + P_{S_{t+1}}^e}{P_{S_t}(1+\delta_1)} = 1 + i_t + \phi_s ,$$

where i_t is a nominal rate of interest, $P_{S_{t+1}}^e$ is the expectation, conditioned on information available during time period t , of the selling price of a single-detached house in time period $t+1$, δ_1 denotes selling costs, depreciation, property taxes, repairs and operating expenses (e.g., heat, electricity) as a fraction of the selling price, and ϕ_s is the risk premium required by investors in single-detached housing (relative to the return available on fixed-income bonds).

The LHS of Condition (4) is the expected (gross) one-period (nominal) rate of return from investing in single housing in time period

t . Rearranging terms and solving expression (4) for w_t yields

$$(5) \quad w_t = P_{S_t} (1 + i_t + \delta_S) - P_{S_{t+1}}^e,$$

where δ_S equals $\delta_1 + \phi_S$ and the approximation becomes exact as the length of the holding period approaches zero.

The RHS of Equation (5) defines the user cost, S_t . In the regressions reported later actual realizations for $P_{S_{t+1}}$ are used in computing values for S_t in place of expectations term $P_{S_{t+1}}^e$. That is, ex post, realized values for the user cost are used in place of ex ante, expected values as the dependent variable in regressions of Equation (3). This substitution introduces an additional stochastic term in the RHS of Equation (3), equal to the difference between expected and realized values for the user cost. Since this stochastic term is an error in an expectation, it must be orthogonal to all of the regressors appearing in Equation (3), which belong to the period t information set. Using ex post realizations for S_t in place of ex ante expectations should have little effect on the parameter estimates.

In order to compute values for S_t , some a priori assumptions must be made regarding the parameter δ_S .

Our reasoning here is that the δ_1 component of δ_S should be in the range of 0.04 to 0.06 on an annual basis, based on an annual depreciation rate of 2 per cent and assuming that annual property taxes, operating costs, etc. run somewhere between 2 to 4 per cent of the price of a house. We have little guidance for selecting a value for the ϕ_S component, other than the observation that the risk premium accorded to

equity returns has averaged in the neighborhood of 6 per cent per annum over the past century. (Grossman, Melino and Shiller (1987)).

Investment in single-detached housing is often highly levered and may be more or less risky than investment in equities. A reasonable value for ϕ_S is likely to fall within the broad range of 0.00 to 0.10 per annum.

Our approach is to allow the data to select the best value for ϕ_S by computing alternative measures for S_t based on differing values for this parameter. A choice among the different measures may be made on the basis of which alternative achieves the best "fit" in regressions of Equation (3). Four alternatives are considered in what follows. These are labelled $S01_t$, $S02_t$, $S03_t$, $S04_t$, respectively, where the numerical suffix refers to the quarterly value for δ_S used in computing the corresponding user cost. For example, in computing $S03_t$, the value for δ_S is set equal to 0.03 in each quarter, corresponding to an annual value of 12 per cent. More specifically $S03_t$ is computed as

$$(6) \quad S03_t = P_{S_t} (1 + i_t + 0.03) - P_{S_{t+1}},$$

where the interest rate, i_t , is measured in decimal units at quarterly rates.

Dropping the numerical suffix for now, the equilibrium price of a single-detached unit is determined by using the law of iterated expectations to solve Equation (5) forward for P_{S_t} :

$$(7) \quad P_{S_t} = E_t \sum_{j=0}^{\infty} \left[\frac{w_{t+j}}{\pi^j (1 + i_{t+s} + \delta_S)} \right]$$

where E_t denotes the expectations operator, conditioned on the information set of time period t .

An estimable approximation to equation (7) is obtained by utilizing the earlier finding that $\frac{w_{t+j}}{CPI_{t+j}}$ is equal to the exponential of the R.H.S. of Equation (3). Observe that the variables appearing in the R.H.S. of Equation (3) are trend-dominated variables (as opposed to being highly cyclically-volatile variables).

Assuming that γ_S is close to 1.0 (rapid stock adjustment), the j^{th} term appearing in the summation in Equation (7) is likely to be well approximated by

$$(8) \quad \left[\frac{w_{t+j}}{\pi^j (1 + i_{t+s} + \delta_S)} \right] = CPI_t \frac{\left[e^{\frac{b_0}{b_2}} \left(\frac{K_S}{N_t} \right)^{\frac{1}{b_2}} \left(\frac{Y_t}{N_t} \right)^{\frac{b_1}{b_2}} \left(\frac{R_t}{CPI_t} \right)^{\frac{b_3}{b_2}} \right] (1+g)^j}{\pi^j (1 + r_{t+s} + \delta_S)}$$

where g is the expected rate of growth of the (real) term in square brackets and

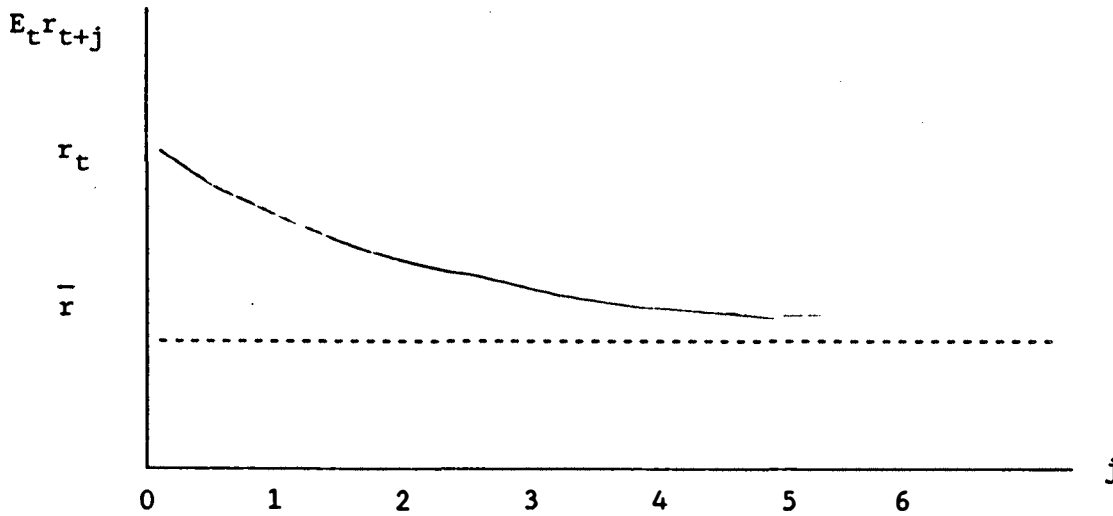
$$(1 + r_{t+j} + \delta_S) = \frac{CPI_{t+j}}{CPI_{t+j+1}} (1 + i_{t+j} + \delta_S),$$

so that r_{tj} is the (ex post) real rate of interest in time period $t+j$.

It is consistent with both economic theory and empirical observation (Klemkosky and Lasser (1985)) to suppose that investors expect real interest rates to exhibit a mean-reverting behaviour. That is, investors view the real rate of interest to be a constant, \bar{r} , in the long run, and expect deviations of r_t from \bar{r} to vanishing monotonically over time in a manner consistent with Figure 1.

Figure 1

Hypothesized Behaviour of Expected Future
Real Rates of Interest



Coupling this hypothesis concerning real interest rate expectations with the approximation described in (8) implies that the expectations term in Equation (7) may be represented by

$$(9) \quad P_{S_t} \approx CPI_t \frac{\left[e^{\frac{b_0}{b_2}} (K_{S_t}/N_t)^{\frac{1}{b_2}} (Y_t/N_t)^{-\frac{b_1}{b_2}} (R_t/CPI_t)^{-\frac{b_3}{b_2}} \right]}{\bar{r} + \delta_S + g} \left[\frac{1 + \bar{r} + \delta_S}{1 + \bar{r}_t + \delta_S} \right]^{\gamma'}$$

where $\gamma' (>0)$ is a parameter relating to the speed with which investors expect r_{t+j} to approach \bar{r} . (Low values for γ' imply rapid adjustment; high values imply slow adjustment.)

Natural logarithms of equation (9) yield an estimable equation from which estimates of the parameters b_1 , b_2 , b_3 and γ' are recoverable.

$$(10) \quad \ln(P_{S_t}/CPI_t) = B_0 + \frac{1}{b_2} \ln(K_{S_t}/N_t) - \frac{b_1}{b_2} \ln(Y_t/N_t) \\ - \frac{b_3}{b_2} \ln(R_t/CPI_t) - \gamma' r_t + V_t.$$

The disturbance term, V_t , appearing in equation (10) arises because of errors in Approximation (9). The intercept, B_0 , is a composite of the logarithms of terms involving $(\bar{r} + \delta_S)$. Estimates of the parameters B_0 , δ_S , and \bar{r} are not recoverable from regressions of the equation. Lastly, r_t enters unlagged by virtue of the approximation

$$\ln(1 + r_t + \delta_S) \approx r_t + \delta_S.$$

Equation (10) provides an alternative to Equation (3) as an equation that can be taken to the data to estimate parameters b_1 , b_2 and b_3 .

Starts of Single-Detached Units

The stock of single-detached units is determined by

$$(11) \quad K_{S_t} = (1 - d_S)K_{S_{t-1}} + \sum_{j=2}^J w_j HSS_{t-j} ,$$

where d_S is the (quarterly) net rate of removal from the existing stock due to fire, conversions, demolition, etc., and HSS_{t-j} is new single-unit housing starts in time period $t-j$.

Construction of a new single-detached unit takes from 2 to J quarters from start to completion. The parameters w_j denote the fractions of units started in time periods $t-j$ that are completed in time period t . Estimates for w_j are to be obtained from regressions of Equations (11), subject to the restriction

$$\sum_{j=2}^J w_j = 1.0 .$$

A value for the net removal rate, d_S , may be obtained from data on end-of-period values for K_{S_t} and quarterly data on completions of single-detached units. (Completions are not explicitly modelled here; but data for completions have been used to interpolate published data for end-of-year values for K_S into the quarterly stock series used in the empirical portion of this study.) A value $d_S = 0.000314$ is implied by the stock and completions data spanning the twenty year time period from the 1961 census to the 1981 census.

This value is considerably smaller than any reasonable estimate of the rate of economic depreciation and suggests that the Canadian housing stock has been "well maintained". That is, the low rate of net removals

suggests that Canadian home owners have made repair and maintenance expenditures at levels that nearly offset economic depreciation.

The quarterly value for d_S converts to a net removal rate of 0.125 per cent per year on an annual basis. This figure is somewhat smaller than the 0.36 to 1.36 per cent per annum range estimated by Leigh [1980] for the U.S. housing stock.

New single unit housing starts are assumed to be supplied to the market by competitive producers. The marginal revenue derived from starting a marginal unit in time period t is the present discounted value of the proceeds from the sale of one unit at the time of completion. We ignore differences between current and near-term future prices here and express this marginal revenue as

$$MR_t = P_S / (1 + i_t)^{\bar{j}},$$

where \bar{j} is an average time to completion.

Real marginal costs in time period t are assumed to be increasing in per capita starts and may also be increasing in the square of the change in per capita starts between time periods t and $t-1$. Expressing real costs as a function of per capita starts allows population to serve as a proxy for the scale of capacity in this industry. Including the square of changes in starts allows for the possibility that there may be "adjustment costs" associated with changes in the level of production. Topel and Rosen [1988] found small but significant adjustment costs in a study of the single-detached sub-market of the U.S. residential housing market.

What the nature of adjustment costs in this industry might be is a

matter of pure speculation on our part. It may be that large adjustments in the level of starts from one quarter to the next involve significant hiring or lay-off costs in regard to construction labour. Whatever the source, the existence or non-existence of adjustment costs is an empirically testable proposition. Our representation of the real marginal costs of producing single units is

$$(12) \quad MC_t = C_0 + C_1 (HSS_t/N_t) + C_2((HSS_t/N_t) - (HSS_{t-1}/N_{t-1})) ,$$

where the parameter C_2 is greater than (equal to) zero as adjustment costs do (do not) exist.

Equating marginal costs with real marginal revenues and adding a stochastic disturbance term yields the estimable equation, expressed in approximate linear form, as

$$(13) \quad HSS_t/N_t = C'_0 + \frac{1}{C_1 + C_2} [P_S / CPI_t - j i_t + C_2 HSS_{t-1}/N_{t-1}] .$$

Estimates of all of the parameters appearing in Equation (13) are recoverable from regressions of the equation.

3. The Sub-Model for Multiple Housing Units

The sub-model for multiple units makes explicit allowances for some of the possible effects of rent controls in this part of the housing market. In order to model these effects we have attempted to quantify certain features of the Canadian experience. By and large the quantification consists of formulating various dummy variables relating to historical events and including these variables in various of the sub-

model's equations. In doing this we have been guided both by economic theory and a sense of what is "reasonable".

The quantification attempts focus on the rent control experiences of British Columbia and, to a greater extent, Ontario. The experiences of Quebec and the remaining provinces have been ignored -- in the former case because Quebec's rent review process has been continuously in effect since WWII. The remaining provinces are ignored simply for reasons of scale.

Rent controls were first imposed in B.C. in October 1974. They remained in effect in one form or another until they were terminated in July 1984.

Ontario first imposed rent controls in July 1975 at the behest of the federal government. The controls were originally linked to the establishment of the federal Anti-Inflation Board and were scheduled to self-destruct in July 1977. In 1977 the controls were "temporarily" extended until the end of 1978. In October 1978 the extension was lengthened, and the controls were made permanent in June 1979. A further significant event occurred in November 1982 when the Ontario rent controls were tightened to disallow the pass-through into rents of "cost" increases due to certain types of rollovers, building swaps and artificial re-financing of mortgages at much higher interest rates.

This brief historical account of the experiences in B.C. and Ontario helps to provide some understanding of our attempts at quantification. We employ four dummy variables pertaining to rent controls in various specifications of the multiple sub-model. These are identified as D1, D2, D3 and D4, and precise definitions of the dummy variables are given

in the variable listing in Section 6.

Variable D1 is essentially a graduated on-off switch. It has a value equal to zero when neither B.C. nor Ontario imposes controls. It has a value of 0.24 when controls are on in B.C. but off in Ontario; a value of 0.76 in the opposite scenario; and a value of 1.0 when controls are on in both provinces. The figure 0.24 is the fraction of population of Ontario and British Columbia that was resident in B.C. in June 1981.

Variable D2 is equal to the four-quarter rate of change in the CPI prior to the imposition of controls in Ontario in 1975:3. In 1975:3 and after, this variable has a value equal to the maximum allowable annual rate of increase in rents permitted under Ontario's rent controls. For example, Ontario imposed a ceiling of 8% in annual rent increases from 1975:3 to 1977:3. The value of D2 is 0.08 over this time interval.

Variable D3 is another on-off switch. It has value of zero prior to 1982:4 and a value of 1.0 in 1982:4 and after. The dummy variable is turned "on" at the point in time when Ontario tightened its controls regarding financing costs.

Variable D4 is yet another on-off switch. We would like this variable to turn "on" at the point in time when builders in Ontario first came to realize that rent controls were going to be made permanent. Permanency is likely to have an adverse effect on incentives and the confidence of owners of apartment buildings. Permanent rent controls imply that rental units constructed today are likely to be subject to controls at some point in the future. This seems almost certain to have adverse effects on new multiple starts.

The realization that Ontario's controls were going to become permanent probably preceded the official announcement to that effect in June 1979. Builders must have seen the handwriting on the wall prior to this announcement but there is no way of knowing when this occurred. One guess is that the recognition occurred some time near the date of the second "temporary" extension of Ontario's rent controls in October 1978. Consequently, dummy variable D4 is constructed with a value equal to zero prior to 1978:3 and a value equal to 1.0 in 1978:3 and afterwards.

The Demand for Occupied Multiple Units

The following specification for the desired stock of occupied multiple units is a direct analogue to the discussion presented in the preceding sub-section pertaining to the stock demand for single houses:

$$(14) \quad \ln(O_t^* K_{m,t-1} / N_t) = a_0 + a_1 \ln(Y_t / N_t) + a_2 \ln(R_t / \text{CPI}_t) \\ + a_3 \ln(S_t / \text{CPI}_t) + \epsilon_{m,t}$$

where O_t^* is the desired occupancy rate during quarter t , $K_{m,t-1}$ is the actual stock of multiple units at the end of quarter $t-1$, $\epsilon_{m,t}$ is a stochastic disturbance and the restrictions imposed on the parameters are

$$a_1 \geq 0; \quad a_2 \leq 0; \quad a_3 \geq 0.$$

The stock variable K_M appears with a lag in Equation (14) because of the way in which occupancy rates are determined. CMHC conducts a semi-

annual survey of vacancies in apartment buildings. We have interpolated the survey results to obtain a quarterly time series on vacancy rates, denoted v_t . The occupancy rate in any quarter is computed as

$$O_t = 1.0 - v_t/100\% .$$

CMHC's survey excludes apartment buildings completed but unoccupied within three months of the date of the survey. Hence, the product $O_t \cdot K_{M_{t-1}}$ is an estimate of the number of occupied multiple units during time period t .

In a free market setting quarter-to-quarter movements in occupied units can be modelled as a partial adjustment in response to discrepancies between desired occupancies and actual occupancies during the previous period. The existence of binding rent controls may inhibit the adjustment process. Would-be occupants may be unable to secure accommodation at prevailing market rents, so that queuing occurs and the speed of the partial-adjustment mechanism is slowed. We model this by allowing the speed of adjustment parameter to be time-varying. Specifically, we hypothesize:

$$(15) \quad \Delta \ln(O_t K_{M_{t-1}} / N_t) = \gamma_{M_t} [\ln(O_t K_{M_{t-1}} / N_t) - \ln(O_{t-1} K_{M_{t-2}} / N_{t-1})] ,$$

where γ_{M_t} satisfies

$$(16) \quad \gamma_{M_t} = \gamma_{M_0} (1 - a_4 D1_t) .$$

The parameter γ_{M_0} denotes the speed-of-adjustment parameter in

a free market. The existence of rent controls (associated with values for $Dl_t > 0$) reduces the speed-of-adjustment whenever $a_4 > 0$.

Equation (15) is an estimatable relationship from which estimates of parameters a_0, a_1, a_2, a_3, a_4 are recoverable. Estimates of a_4 , subject to a non-negativity restriction, can provide evidence of whether Canada's experience with rent controls has restricted the availability of rental accommodation.

The inclusion of the dummy variable, Dl , as a determinant of the speed-of-adjustment parameter via Equation (16) is a reasonable way to model the effects of rent controls on the stock demand for rental housing. What cannot be adequately modelled, however, is the likely distorting effect of binding rent controls on the measured value of the vacancy rate (and its counterpart: the occupancy rate). The survey methods employed to estimate the economy-wide value of v_t ignore the possibility that binding rent controls may result in queues of would-be renters who are unable to find accommodation at prevailing rents.

The measured value of v_t may not provide a reliable estimate of the state of excess supply in the rental housing market whenever rent controls cause queuing in some parts of the country. The stock demand for multiple units is apt to appear lower than it actually is. When such conditions prevail, an increase in the stock supply of multiple units is apt to result in a nearly one-to-one increase in occupancies as queues are shortened in response to an increase in available accommodation.

What all of this means is that while Equation (15) provides a reasonable representation of the response of stock demands for multiple units in response to changes in incomes and prices, it can not be

expected to fully capture the response of occupancies/vacancies to changes in the stock supply of multiple units in the presence of binding rent controls. Since the problem is inherent in the measurement of vacancies, it has no easy remedy. In using an estimated version of Equation (15) to perform simulations of the impacts of the CMHC rental subsidy programs on occupancy/vacancy rates (as is reported in Appendix D) we will have to be prepared to modify predictions of the equation to allow something close to a one-for-one response of occupancies to simulated changes in the stock supply of multiple units during time periods over which rent controls were operative.

The Behaviour of Rents

In a free-market environment rents might be expected to respond to changes in operating costs and vacancy rates in a manner similar to the workings of an expectation - augmented Phillips curve. Operating costs are assumed here to move in proportion to changes in consumer prices. The imposition of binding rent controls would reduce, and possibly eliminate, the ability of rents to move in response to the market conditions signalled by vacancies. Our representation of this is

$$(17) \quad \ln R_t - \ln R_{t-4} = (1 - \theta_t) [\ell_1 (\ln \text{CPI}_t - \ln \text{CPI}_{t-4}) + \ell_2 (\bar{v}_t - v^*)] \\ + \theta_t D2_t + \epsilon_{R_t},$$

where a bar over a variable denotes a four-quarter moving average and θ_t is described by

$$\theta_t = l_0 \overline{D1}_t .$$

The term in square brackets in (17) depicts the behaviour of rents in a free-market. The parameter v^* denotes a normal, or natural, vacancy rate which will be maintained on average by movements in rents over long time periods. Parameter l_1 should have a value close to 1.0 and parameter l_2 should be negative in sign.

The variable θ_t denotes the fraction of the market subject to rent controls between time periods $t-4$ and t . The actual change in rents over this four-quarter interval is a weighted average of the free-market change and the allowable change in rents in the controlled segment of the market, as measured by $D2$.

Equation (17) is estimatable and all of its parameters are recoverable.

Multiple Stocks and Housing Starts

The stock of multiple units is related to past starts via

$$(18) \quad K_M_t = (1 - d_M)K_M_{t-1} + \sum_{j=1}^J \lambda_j \cdot HSM_{t-j}$$

where d_M is the net removal rate, HSM_{t-j} is multiple starts in quarter $t-j$ and the parameters satisfy

$$\sum_{j=1}^J \lambda_j = 1.0; \quad \lambda_j \geq 0, \quad \text{all } j .$$

The value for d_M , estimated from annual stock and completions

data, is 0.000519 per quarter.

The behavioral motivation for the specification of an equation determining multiple starts (per capita) is similar to that described for the singles sub-model. The real marginal cost of producing multiple starts, in time period t , is assumed to be increasing in HSM_t and, possibly, in $(HSM_t - HSM_{t-1})$, reflecting possible adjustment costs.

The difficulties in this market lie with modelling the marginal revenues received by builders of multiple units. There exists no available data series for the selling prices of newly constructed multiple units. We must find some proxy measure for this price. In addition, the possible distorting effects of rent controls must be somehow included in the specification. Finally, the specification must enable us to estimate the effects of the various CMHC subsidy programs on new starts.

With regard to the problem of finding a price proxy, we have at our disposal three candidates. The first is the constructed data series, PV1, from Appendix A on the capitalized values of The Subsidies Delivered by the Federal Rental Initiatives. This series measures the capitalized value of the stream of after-tax earnings associated with a prototypical apartment building. It may serve as a reasonable proxy but is likely to overstate the true selling price of an apartment unit during time periods in which owners fear that future rents may become subject to rent controls.

A second candidate is the PV2 series, also from the preceding Appendix. This series is similar to PV1, but includes the capitalized value of tax savings associated with the M.U.R.B. program. Since all

rental units were eligible for MURB certificates, the PV2 series may come closer to measuring market price than PV1 but is also likely to overstate price during periods of rent controls.

The third candidate is the rent variable, R_t . This can serve as a reasonable proxy for the selling price of new apartment buildings whenever future rents are expected to change smoothly over time. As a price proxy the rent variable may capture some aspects of rent controls slightly better than the PV variables.

Our strategy here is to estimate an equation determining multiple starts for each of the three alternative proxy measures for price. A fourth alternative is also to be estimated -- this one a "reduced-form" equation in which real income appears in place of the price variable.

Included in each of the alternative equations will be a rate of interest, one or two of the dummy variables associated with rent controls, a set of variables designed to isolate the effects of the various CMHC subsidy programs on multiple starts, and the lagged value of starts.

There is little theoretical guidance to use in specifying the way in which rent controls might affect starts. In principle, rent controls might depress the selling prices of apartment buildings and thereby be captured in prices. Among the various proxies, only the rent variable is apt to embody this effect. The PV1 and PV2 variables were constructed using fairly simple extrapolation techniques that ignore the possibility that future rents may be subject to controls. The use of dummy variables D3 and D4 as additional regressors in regressions involving PV1 and PV2 and in the reduced-form regression is designed to capture both the direct

effects of controls on rents as well as any additional expectations effects associated with the introduction or tightening of rent controls.

Effects of the various subsidy programs on multiple starts are to be estimated by including in each regression variables that measure the capitalized value of the incremental profits derived by a prototypical apartment building under each program. These variables are designated ARP_t , $CRSP_t$, LD_t and $MURB_t$, where the mnemonics are obvious. Details regarding the construction of these variables are given in the variable listing in Section 6.

It is worthy of mention that when either PV1 or R is used as the price proxy, each of the program variables measures incremental capitalized values, relative to PV1. The same representations are used in the reduced-form regression. In regressions using PV2 as the price proxy, ARP_t , $CRSP_t$ and LD_t measure specific program increments relative to PV2. The $MURB_t$ variable is set equal to a dummy variable in this case, and has a value equal to 1.0 when the M.U.R.B. program was in effect and zero otherwise.

Two problems associated with the subsidy program variables seem likely to arise in the empirical investigation. The first is multicollinearity. Given the large numbers of variables that will be used as regressors, it will be surprising if significant coefficients can be identified for each of the ARP, CRSP, LD and MURB variables. It will likely be necessary to impose some restrictions on these coefficients.

An obvious restriction is to constrain the coefficients on the first three to be identical by using the sum ($ARP_t + CRSP_t + LD_t$) as a single regressor. These three programs did not overlap, nor were they universal

in the same sense as the M.U.R.B. program. Imposing the stated restriction is likely to prove to be an efficient means of obtaining reliable estimates of their net impacts on starts. The restriction still permits the net effects of the MURB variable to be independently estimated.

A second problem associated with estimating the effects of the subsidy program is more severe. Each of the programs was initially motivated by poor economic performance in some aspect of the rental market. With the exception of the Limited Dividend Program, the programs were introduced during periods of locally low multiple starts. This is almost certain to lead us to under-estimate the impacts of the programs. In effect, there is a simultaneous equations bias likely to be operative in the regressions, and we are missing the "other equation" -- one that describes the reaction function of the policy authorities.

Our strategy is to postpone consideration of the simultaneity problem in a "first round" of estimation. The first round consists of performing regressions using multiple starts as the dependent variable and experimenting with the various price proxies, subsidy variables and rent control dummies as regressors. After having narrowed down the possibilities by eliminating a number of alternative specifications, we will consider the simultaneity issue.

4. Empirical Results: The Sub-Model for Single-Detached Houses

The empirical results are reported here in the order of the discussion of the previous two Sections. A complete description of the variables used in obtaining the results appears in a listing in Section

6.

For the most part, the equations described in the text were fitted over the sample period 1970:1 to 1987:4. The length of the sample period was dictated by the availability of data for certain key variables; in particular, for the price of single-detached houses and the vacancy rate for multiple units.

Unless otherwise indicated, estimated equations were fitted by Ordinary Least Squares regressions.

Demand for Single-Detached Houses

Table 1 reports results for regressions of Equation (3), utilizing the four alternative measures for the user cost variable previously described. This equation had to be fitted in linear (as opposed to log-linear) form because constructed values for each of the user cost variables had at least one negative value over the sample period.

Recall that the S01 to S04 alternatives are ex post realized values for the user cost of owner-occupied single-detached houses. A large unanticipated increase in the selling price of single units can cause the ex post user cost to be negative, even if the ex ante, expected value is always positive. This apparently occurred over the sample period, as values for each of S01 to S04 were sometimes negative. Ideally, one would like to exclude extreme observations of ex post realization from the data set. One way to do this would be to simply discard all observations above and below, say, two standard deviations from the mean for each alternative. This would eliminate roughly 5 per cent of the observations. We did not employ this truncation because the number of

observations is already quite small.

The regression results reported in Table 1 are not very satisfactory. Both the income and rent variables obtain coefficients with signs inadmissible on theoretical grounds in Regressions (1) to (4). The coefficients on the stock variables have the anticipated signs in these regressions but imply an implausibly low speed of adjustment in the range of 0.10 to 0.12 per quarter.

Regressions (5) and (6) report the effects of eliminating the lagged stock variable from the specification using the S04 measure for the user cost. Dropping $(K_S/N)_{-1}$ effectively constrains the speed of adjustment parameter, γ_S , to equal 1.0. This does not cause an appreciable worsening in the fit of the equation, but neither does it reverse the signs of the income and rent variables--even after making a correction for first-order serial correlation of the residuals.

Dropping the rent variable yields a "correct sign" for income in Regression (7), but the stock variable has an incorrect sign. Regression (8) applies a first-order serial correlation correction to the specification of Regression (7) but fails to yield acceptable parameter estimates.

The difficulties encountered here prompted us to turn our attentions to Equation (10)--the forward solution for the price of single-detached houses. This equation contains all of the demand parameters embedded in Equation (3) and provides an alternative for estimating the values for these parameters. Further, some fitted version of the equation determining price is essential for later simulations of the housing sub-model, whereas Equation (3) is not.

Empirical results for equation (10) are reported in Table 2 and these proved to be much more satisfactory.

The variable identified by DCHOSP in the table is a dummy variable that has a value of 1.0 from 1982:3 to 1983:2 and a value of 0.0 elsewhere. The 1982:3 to 1983:2 period corresponds with the duration of the Canadian Home-ownership Stimulation Plan (CHOSP), which provided a grant of \$3,000 to buyers of new houses and first-time buyers of existing houses. Including this dummy variable in the regressions allows for the possibility that the grants issued under CHOSP were capitalized (in whole or in part) in the prices of single-detached houses. The various regressions reported in Table 2 show no evidence that this possibility was realized.

What the regressions do indicate is that the price of single-detached houses is positively related to real disposable income and negatively related to both the existing stock of single houses and the real rate of interest.

When it is included in the regressions, the rent variable obtains a coefficient with the "incorrect" sign. We interpret the failure of the rent variable to obtain a positive coefficient as lack of evidence in support of the proposition that there is significant cross-substitution of demand between single and multiple housing. Regressions (4) to (6) constrain the parameter on (R/CPI) to be zero--the lowest value consistent with economic theory.

The preferred regression from Table 2--and the one which we shall use in later simulation exercises--is Regression (5). This equation has an implied income elasticity of demand for single-detached housing of

0.46 ($= 1.7256/3.7195$). The implied price elasticity of demand is -0.27. These are both short- and long-run elasticities. The regression also implies that a one percentage point increase in the real rate of interest will reduce the selling price of single-detached houses by 1.2 per cent.

The estimated value for real interest rate impact seems somewhat low, though there is little competing evidence with which to compare this finding. The estimated price elasticity of demand also seems to be somewhat low but, once again, there is little in the existing literature with which to compare this figure. The one estimated value for which comparisons are readily available is the income elasticity of demand.

Sparks [1986] estimated the long-run income elasticity of demand for Canadian single-detached housing stock to be 0.67. Oksanen [1966] obtained a value of 0.5. Carliner [1972] found an income elasticity of demand for owner-occupied housing in the United States to be between 0.6 and 0.7. The 0.46 value implied by regression (5) is certainly not out of line with these previous findings.

One troublesome feature of regression (5) is its low value for the Durbin-Watson Statistic. The Durbin-Watson statistic is a diagnostic designed to indicate the absence or presence of first-order serial correlation in the residuals of a fitted equation. Attempts to use standard techniques to correct for this problem (reported in Regression (6)), surprisingly, did little to the value for the Durbin-Watson statistic. This suggests that the residuals from Regression (5) follow some higher order process.

One possible explanation for these results is a time aggregation

bias. If the P_S variable adjusts continuously through time in response to market conditions, then discrete time estimation of an equation like Regression (5), which uses quarterly averages of the variables, can yield residuals that follow a first-order moving average process. This phenomenon has been routinely encountered in empirical studies of asset prices (e.g., Melino, Grossman and Shiller [1987]).

A moving-average disturbance will yield a low Durbin-Watson Statistic, and a first-order serial correction procedure is not an appropriate correction. What is required, instead, is a correction procedure designed to detect an MA(1) residual error. However, this is a complicated procedure and one that is beyond the capabilities of standard regression packages (including the one utilized in this study).

An investigation of the moving-average properties of the residuals from Regression (5) is a possible topic for future research. For now, Regression (5) is adopted as is, both because it contains reasonable estimates of the parameters and because its overall fit ($\bar{R}^2=0.72$) is reasonably good.

Supply of Single-Detached Houses

Estimating Equation (11), relating the stock of single units to single starts proved a straight-forward exercise. The results are reported below.

$$\begin{aligned}
 K_S &= 0.999686 K_{S-1} + \frac{0.6661}{(1.99)} HSS_{-2} + \frac{0.1634}{(0.34)} HHS_{-3} \\
 &\quad + (1.0 - 0.6661 - 0.1634)HHS_{-4}
 \end{aligned}$$

Sample Period : 1962:3 to 1987:4
 Standard Error Regression: 13.6 (thousands of units)

Regressions involving Equation (13) for new starts of single-detached units are reported in Table 3. The results are quite satisfactory. Movements in the price of single-detached houses prove to be a reliable predictor of quarter-to-quarter movements in single starts.

Regressions (1) to (6) report variants of the basic specification using real or nominal interest rates as alternative measures of financial pressures on builders. The evidence supports the use of only the nominal rate of interest. The evidence also suggests that the CHOSP program had a predominantly positive but statistically insignificant impact on single starts during the 1982:3 to 1983:2 period. Both of these findings contrast with results obtained by Sparks [1986]. Sparks found that both nominal and real interest rates had significant effects on new single starts over a 1965:1 to 1984:4 sample period. Additionally Sparks found a significant stimulus associated with CHOSP.

Sparks' study did not employ the P_S variable but used in its stead real income and the lagged stock of houses as proxies for market conditions. We were able to approximately replicate Sparks' results using a 1965-1984 sample period and our own measures for real income and stocks. The replication was not exact because of revisions to the data series that have occurred since Sparks completed his study.

One thing we did find was that using Sparks' specification, estimates of the parameters were quite sensitive to the choice of sample period. Extending the sample period to cover 1962:2 to 1987:4 yielded quite different parameter estimates. (These results are not reported

here but are available upon request.) One finding of particular interest is that the dummy variable for CHOSP becomes much less significant when the longer sample period is used.

In any event, the preferred specification was reduced to a choice between Regressions (5) to (8). Our choice is Regression (5) which exhibits a small, positive impact associated with CHOSP. While the coefficient on the dummy variable in Regression (5) is not statistically significant at a 5 per cent (even a 10 per cent) level of confidence, it is nonetheless in excess of its standard error. Thus DCHOSP contributes positively to the goodness-of-fit and Regression (5) has the lowest standard error of regression among the contenders.

Observe that Regression (5) has a correction for first-order serial correlation. Regression (6) does not have this correction but includes the lagged dependent variable as an additional regressor. Regression (6) appears to support the existence of adjustment costs in the construction of single-detached homes. However, the large significant coefficient obtained by $(HSS/N)_{-1}$ may simply be the result of serial correlation in the disturbance term. It might be incorrect to attribute the significance of the lagged dependent variable as indicating costs of adjustment in this case.

Our reasoning is that if there are indeed adjustment costs in this industry, Regression (6) ought to fit the data better than Regression (5). This is not the case; nor is it the case when comparisons are made between Regressions (2) and (3) and Regressions (7) and (8). The choice of Regression (5) as the preferred result depicts the construction of single-detached houses as occurring in an industry that does not face

costs of adjusting production.

In contrast, Topel and Rosen [1988] did find some evidence of adjustment costs in the construction of single units in the United States. However, their estimate of the magnitude of the effect on new single starts is quite small and corresponds more closely with the zero adjustment costs implicit in Regression (5) than with the relative high adjustment costs implied in Regression (6).

The parameter estimates in the preferred equation imply (a) that CHOSP stimulated approximately 22,560 single-detached starts during its four-quarter lifetime, (b) that a one percentage point increase in the mortgage rate of interest would reduce single starts by 8,965 units per year (based on population figures for 1981:2) and (c) that a one per cent increase in the price of single houses relative to the CPI would raise single starts by 1,023 per year (based on population figures for 1981:2). The last figure indicates a price elasticity of supply equal to 1.15 in 1981.

5. Empirical Results: The Sub-Market for Multiple Housing Units

Stock Demand for Multiple Units

Regression results for estimation of stock demand Equation (15) are reported in Table 4. Regressions were also performed on a linear variant of Equation (15). These proved to be inferior to the log-linear results and are not reported here. It is worth mentioning that the linear equations did not reveal evidence of cross-substitution between the demands for multiple and single housing. The user cost of single houses failed to enter the regressions with a significant and positive

coefficient. This finding is consistent with the failure of the rent variable to exert any significant influence on the stock demand for single units. Consequently, the user cost variable was dropped and does not appear in Table 4.

Regression (1) imposes no constraints on the parameters other than those explicit in Equation (15) (with a_3 set equal to zero). That \hat{a}_4 is significantly different from zero is evidence of some slowing in the speed with which renters adjust to discrepancies between desired and actual stocks during the period of rent controls in Ontario and/or British Columbia. Unfortunately, the estimated elasticity of demand with respect to rents, \hat{a}_2 , is positive in this regression and this violates the sign restriction imposed by economic theory.

Regression (2) duplicates the first regression, using a measure of permanent income in place of current disposable income. The results are nearly identical to those of Regression (1), except that the estimate, \hat{a}_2 , is now both positive and significantly different from zero.

Given these results, we felt compelled to impose some restrictions on the parameters. Some rationalization for imposing constraints here can be made on the basis that our attempts to model the effects of rent controls, while plausible, are nonetheless crude. The dummy variable, $D1$, used to introduce rent controls into Equation (15), is a constructed variable. There is nothing to guarantee that $D1$ is the "appropriate" variable to introduce here.

Our first modification was to constrain the parameter a_1 to have a value 0.5. This was based on the reasoning that a_1 represents the long-run income elasticity of the stock demand for multiple units. There

is little reason to expect a_1 to differ from the long-run income elasticity of demand for single units, and we have considerable prior information (cited in the previous Section) concerning the value of that elasticity.

Constraining a_1 to equal 0.5 did produce the desired effect of causing the estimated value of a_2 to be negative. However, it also yielded an estimate for γ_{m_o} , the free-market speed of adjustment, that was slightly greater than 1.0. Since a value for $\gamma_{m_o} > 1$ makes no theoretical sense, we dropped the constraint on a_1 and imposed a constraint $\gamma_{m_o} = 1$. The results of the latter are reported as Regressions (3) and (4).

Regression (3), which utilizes current--as opposed to permanent--income, is our preferred regression equation. It implies long-run income and price elasticities of demand of 0.57 and -0.13, respectively. The latter seems low, but is not far out of line with the corresponding estimate for the single-detached sub-market.

The estimated value for a_4 in Regression (3) is uncomfortably large, for it implies a large and abrupt reduction in the speed of adjustment with the imposition of rent controls in just two of Canada's provinces. This is an area in which future research might yield more plausible results.

The Adjustment of Rents

Table 5 summarizes the results of regressions involving Equation (17). Regression (1) shows unconstrained estimates of the parameters in this equation. The negative and statistically significant value obtained

for \hat{l}_2 provides some evidence that the rate of change of rents in a free-market is responsive to the vacancy rate. The statistically significant estimate for l_0 is an indication that the imposition of rent controls inhibits the free-market adjustment process.

The peculiar feature of Regression (1) is the estimated value for l_1 , the coefficient on rates of change of consumer prices. A negative value here is both surprising and nonsensical. We suspect that this result may be an artifact of our attempts to model the effects of rent controls using constructed dummy variables.

Equation (17), like Equation (15) before it, imposes considerable structure on the variables appearing in it. If we do not measure some of these variables accurately--and this is almost surely the case for the dummy variables--then this shows up in the parameter estimates. To make the best of this situation, it is probably reasonable to utilize as much prior information concerning the parameters as is available, and we do have some reliably-based priors concerning some of these parameters.

It would make little theoretical sense for the rate of growth of nominal rents to deviate from the rate of growth of prices over the long-run, at least in a free-market environment. Consequently, it does not seem unreasonable to impose the constraint $l_1=1.0$ on Equation (17). This is done in Regression (2), which, unfortunately, yields implausible estimates for parameters l_2 and l_0 .

Regression (3) imposes $l_1=1.0$ and $l_0=0.47$, on the reasoning that the dummy variables relating to rent controls were constructed on the basis of the rent control experience of Ontario and British Columbia, in which 47 per cent of the Canadian population resided in 1981. This

regression is performed as a sort of diagnostic check on the dummy variables. If they actually measure what we think they are measuring, then the parameter l_0 should be in the neighborhood of 0.47. The bizarre values obtained by the free parameters in Regression (3) appear to confirm that the problems here do reside with our attempts to account for the effects of rent controls.

Using the reasoning that the natural vacancy rate in a free market ought to be the average vacancy rate observed over a long time interval, we impose the joint restrictions $l_1=1.0$ and $v^*=2.5$ (per cent) in Regression (4). (The value 2.5 is approximately the sample period average value for v_t .) This yielded a plausible estimate for l_2 and an estimate for l_0 that is not beyond the bounds of what is acceptable.

Regression (5) was performed mostly for curiosity's sake. It combines all three constraints previously discussed. Regression (6) imposes $l_1=1.0$, together with a value -0.02 for l_2 . The -0.02 value is the estimate of l_2 obtained by an Ordinary Least Squares regression of the free-market variant of Equation (18) over the very short sample period 1970:2 to 1974:3.

We are tempted by the superior fit of Regression (6) over Regression (4), but feel that there is not sufficient evidence to support constraining l_2 in deference to constraining v^* . Regression (4) is chosen as the preferred equation for later model simulations.

The Stock of Multiple Units

The fitted equation utilized to update the stock of multiple housing units is given below.

Sample Period: 1962: to 1987:4

Standard Error of Regression: 5.454 (thousands per quarter)

$$\begin{aligned}
 K_M = & 0.999481 K_{M-1} + 0.4902 HSM_{-2} + 0.0607 HSM_{-3} \\
 & + 0.2328 HSM_{-4} + 0.0511 HSM_{-5} \\
 & + (1.0 - 0.4902 - 0.0607 - 0.2328 - 0.0511)HSM_{-6}
 \end{aligned}$$

(4.65) (0.46) (1.76) (0.38)

New Starts of Multiple Units

As described in the discussion of the model, there are three possible variables that might be used as proxies for the (unobservable) unit selling price of a rental property. Each of these provides a possible specification for the regression equation. A "reduced-form" expression which substitutes real income and lagged stock in the place of a "price" variable adds a fourth possibility.

Rather than experiment with functional form for each of these possibilities, our strategy was to conduct extensive experimentation using just one of the "price" variables. The deflated value of the base capitalized value series, PVL--from Appendix A, was selected for this task. We performed regressions using this series as the main explanatory variable in order to determine the "best" way to incorporate the dummy variables associated with rent controls and the incremental profit variables associated with the various CMHC subsidy programs. The functional form that emerged from these explanations as "best" was then utilized in the regressions involving the alternative price proxies and the reduced form.

A selection of the regressions performed using the PV1 series is presented in Table 6. The regressions are listed in approximately the order in which they were run. The symbol PV1R denotes PV1 divided by CPI.

Results from Regressions (1) and (2) rapidly convinced us that it would be impossible to obtain meaningful estimates of the individual parameters associated with each of the CMHC subsidy programs. Some aggregation would have to be applied here, and we imposed the aggregation described in Section 3 to combine the Limited Dividend, ARP and CRSP variables. The M.U.R.B. program was left separate.

A number of other discoveries were made with the earlier regressions. Of the dummy variables introduced to capture various aspects of rent controls, only the variable D4 exerted a persistently negative impact on multiple starts across the alternative regressions. This is the dummy variable that has a value of 1.0 in 1978:4 and after--corresponding with our estimate of the time period during which rent controls in Ontario have been "permanent" (or perceived to be permanent). The other rent control dummies performed poorly and erratically and were ultimately dropped from the preferred specification: Regression (11). The failure of dummy variable D3 to exert a persistent negative effect on new starts is especially surprising.

When entered without interaction with the nominal rate of interest, the deflated value of the PV1 series (denoted PV1R in the table) persistently obtained a negative coefficient. The sign of this coefficient is reversed when the interaction term, $PV1R \cdot i$, is included as a regressor, and the fit is always better when this term is included.

(This result is common among all of the alternative proxy price variables.) It should be noted, however, that every regression in which PVLR is interacted with i has an implied "price" elasticity of multiple starts that is negative at or above critical values for the mortgage rate that are persistently exceeded during the sample period. In this respect, none of the regressions reported in Table 6 is truly satisfactory.

Inclusion of the lagged dependent variable as a regressor invariably improves the fit of any variant reported in the table. The fortuitous result that lead us to exclude the lagged dependent variable from the preferred equation for single starts is not operative in the regressions for multiple starts. The preferred regression here includes the lagged value of (HSM/N).

Coefficient estimates for the CMHC subsidy variables are quite unstable across these explanatory regressions. Neither the variable measuring the (deflated) incremental profits associated with M.U.R.B.'s nor the aggregated variable pertaining to the other programs obtains a consistently positive coefficient. Both variables have positive coefficients in the preferred Regression (11), but neither coefficient differs significantly from zero. We are unable to attach a very narrow confidence interval about the estimates of the program effects obtained from this regression.

Before proceeding to the regressions involving the alternative price proxies, it is worthy of mention that the dependent variable in these and the regressions reported in Table 6 is measured in units of seasonally-adjusted quarterly starts per 1 million population. This scale was

chosen as a way of ensuring that the coefficients appearing in the tables do not span an inordinately broad range of decimal places. By way of illustration: The coefficient obtained by the variable D4 in Regression (11) implies that rent controls reduced multiple starts by $(105.1 * N * 10^3)$ units in any quarter after 1977:3. Ignoring the complicating effects of the lagged dependent variable, this works out to approximately 16,500 starts per year.

Table 7 reports the results of utilizing the preferred specification in regressions involving the alternative price proxies and in the reduced-form regression.

Variables measuring the incremental effects of M.U.R.B.'s obtain coefficients that are marginally significantly different from zero in all of the alternatives to PV1. Variables measuring the incremental collective effects of the other CMHC subsidy programs obtain positive but statistically insignificant coefficients. The magnitudes of the coefficients for both the M.U.R.B. and the component subsidies are roughly equal in alternative Regressions (3) and (4). The coefficients for these subsidies in Regression (2) cannot be directly compared with those in the other regressions because the subsidy variables differ here in the ways in which they are measured.

Regression (2), based on the PV2 series, has an implied "price" elasticity that is negative over most of the sample period. Regression (3), based on the rent index, does not have this property. This regression has an implied "price" elasticity that is positive over all realistic values for the nominal rate of interest on mortgages.

Having narrowed down the alternative specifications to those shown

in Table 7, we turn now to the simultaneous equations issue. The crux of this concern is that the introduction of the various subsidy programs were each in part motivated by policy-makers' concern over periods of poor performance in the rental housing market. At least three of these subsidy programs first appeared during a period of declining activity in multiple starts -- a timing coincidence that may cause the impacts of the various programs on new starts to be under-estimated in the regressions reported in Table 7.

The variables MURB and (LD+ARP+CRSP) appearing as regressors in the regressions of Table 7 are not truly exogenous to the model of the housing market. Missing from our analysis up to this point is a description of how the values for these subsidy variables are endogenously determined by policy makers. What is ideally required is a set of policy reaction functions describing how values for MURB, LD, ARP and CRSP were varied by policy makers in response to changing conditions in the market for rental housing.

It is clearly difficult to posit the formal existence of a set of policy reaction functions without a detailed understanding of policy makers' concerns and constraints in the case of each of the subsidy programs. As a relatively simple check to determine whether the omission of policy reaction functions has caused some downward biases in the estimated coefficients of the subsidy variables appearing in the Table 7 regressions, we introduce an additional dummy variable, DD2, into the specifications.

DD2 has a value equal to 1.0 during any quarter in which a subsidy program first appears, or changes in scope over the 1970:1 to 1987:4

sample period. Thus DD2 has values equal to 1.0 in 70:1 (the first appearance of LD's in the sample period), in 74:4 (the start of MURB's), in 75:2 (the start of ARP's), in 76:2 and 78:2 (changes in the scope of ARP's), in 80:4 (the revival of MURB's) and in 82:1 (the start of CRSP's). Variable DD2 has a value of 0.0 elsewhere.

Our reasoning is that the inclusion of DD2 as an additional explanatory variable in the Table 7 regressions should provide an indication of the extent of simultaneous equations bias. If bias is present, DD2 should obtain a negative coefficient, improve the fit of the equations and cause an increase in the estimated coefficients for the MURB and (LD+ARP+CRSP) variables.

All three of the anticipated effects are observed in the regressions which are reported in Table 7a. (The reduced - form Regression (4) is not included in Table 7a since it appeared in Table 7 only for purposes of comparison.)

The "preferred" result in Table 7a is Regression (3a) in which the rent variable, R, serves as the proxy measure of market price. This result is preferred because it is the only regression to exhibit a persistently positive relationship between multiple starts and "price". (Regressions (1a) and (2a) both contain an implied negative relationship between starts and "price" whenever the nominal rate of interest on conventional mortgages in excess of 8.0% -- a circumstance that prevailed throughout the sample period.) Equation (3a) is utilized in the macro-economic simulation exercises described in Appendix D.

In comparison with Regression (3), Regression (3a) has a slightly improved fit and attributes a somewhat larger impact on starts to the

collective subsidies provided by the LD, ARP and CRSP programs. The estimated coefficients for the MURB variable are virtually identical in the two regressions.

The addition of the dummy variable DD2 helps to rectify the simultaneous equation bias but is clearly not a cure-all here. The t-statistic for the coefficient on (LD+ARP+CRSP) increases relative to its value in Regression (3) but remains well below the usual cut-off values associated with statistical significance levels of 5 or 10 percent. Nonetheless, at 0.8851 the point estimate for this coefficient is positive and may be regarded as our single best estimate for the collective effects of the LD, ARP and CRSP programs on multiple starts.

Not reported here are some empirical results based on an effort to deal with the simultaneous equations problem by postulating an explicit policy reaction function relating, collectively, to all of the subsidy programs. Estimation of the parameters in specifications of multiple starts equations similar to Regressions (1)-(3) in Table 7 using this more formal approach did suggest the presence of downward biases in the coefficients pertaining to the MURB and (LD+ARP+CRSP) variables. However, the overall fits of the estimated equations were inferior to Regressions (1a)-(3a). Our decision to proceed with Regression (3a) as the preferred result is reinforced by the finding that the more sophisticated econometric analysis did not generate a better result.

Our subjective assessment of why greater statistical significance cannot be attached to the impacts of the subsidy programs centers on the inadequacies of the various "price" variables to closely proxy for the unobservable selling prices of multiple-unit buildings. The empirical

success of the regressions involving single starts suggests that the singles market is well described by a fairly conventional model rooted in economic theory. Data exist for selling prices of new single-detached units and this variable proved to be the key explanatory variable in the single starts regressions.

Our feeling is that if a price series of comparable quality existed for multiple units, the empirical results for multiple starts would be equally as good as they turned out to be for singles. Not having such a data series is a serious shortcoming, especially over a sample period in which rent controls make the use of proxy price measures even more problematic than would otherwise be the case.

In spite of the limitations imposed by the lack of data, Regression (3a) implies that the various subsidy programs provided considerable positive stimulus to multiple starts over the sample period. The incremental capitalized values measured by the MURB, LD, ARP and CRSP variables yield a much better fitted equation than can be obtained by the next best alternative of simply regressing multiple starts on rents, interest rates and a set of zero/one dummy variables for the various subsidy programs.

For purposes of comparison, the results of this kind of alternative regression are summarized here.

$$\begin{aligned}
 (\text{HSM}/N) = & -274.8 + 712.8 (R/\text{CPI}) + 7.7874 i^*(R/\text{CPI}) - 47.684 D4 \\
 & (0.53) \quad (1.87) \quad (0.51) \quad (-0.26) \\
 & -137.9 \text{ DD2} - 44.508 \text{ DMURB} - 36.449 \text{ DLD} + 223.1 \text{ DARP} \\
 & (-.52) \quad (-0.48) \quad (-0.26) \quad (1.60) \\
 & -124.4 \text{ DCRSP} \\
 & (-1.32)
 \end{aligned}$$

Sample Period: 1970:1 to 1987:4
Method of Estimation: Ordinary Least Squares
Standard Error of Regression: 214.1
Adjusted R² = 0.70
D.W. = 2.01
F(9,62) = 20.1

The variables DMURB, DLD, DARP, and DCRSP are dummy variables for the subsidy programs with values equal to 1.0 when the respective program is operative and a value equal to zero otherwise. The overall fit of this alternative regression is about the same as that of Regression (3a). However the alternative attributes negative net impacts on starts to all but the ARP program. Further, the alternative regression implies that higher nominal interest rates will stimulate more, not fewer starts, starts. This is an obviously inferior result to our preferred regression.

Some indication of the quantitative implications of the preferred Regression (3a) for the impacts of the subsidies on new starts is provided in Tables 8-11. Table 8 reports on an annual basis the numbers of multiple starts attributable to the Limited Dividend program from Regression (3a) for each of the eighteen years of the sample period. For purposes of comparison the numbers of starts attributable to LD's from Regressions (1a) and (2a) in Table 7a are also reported in Table 8. Table 9 shows the starts attributable to the MURB program for each of the three regressions. Tables 10 and 11 do the same for the ARP and CRSP programs, respectively.

The figures in the Tables were obtained from single-equation simulations of each of Regression equations (1a), (2a) and (3a). Each of

the regression equations was solved from 1970:1 to 1987:4 using actual historical values for all its RHS variables except for the lagged values of HSM from 1970:2 to 1987:4. Each equation's solution for HSM was used as the value for lagged starts in computing the next period's solution value. This exercise yielded a time series of predicted values for multiple starts for each of the regression equations.

In order to generate a time series for the estimated effects of, say, the Limited Dividend program, the above procedure was repeated after setting values for the variable LD equal to zero in each time period. This generated a second series of solution values for multiple starts for each equation. Differences between the first series of solution values and the second provide estimates of the net direct impacts of the Limited Dividend program on housing starts. It is these values that are reported on annual basis in Table 8.

Similar procedures were used to generate estimates of the net direct impacts of each of the other subsidy programs.

The figures appearing in Tables 8-11 are referred to as net direct impacts because

- (i) they show the effects of the various subsidy programs on multiple starts, net of any immediate offsets. (Units started under, say the Limited Dividend program, may have displaced some multiple units that would have otherwise been started in the absence of LD's. The figures in Table 8 show the net increments to starts attributable to LD's.)

- (ii) they excluded any offsetting reduction in starts that may have

been indirectly induced by subsequent changes in rents, prices, incomes, vacancy rates, or interest rates. (If multiple starts were higher because of, say, the Limited Dividend program, then market values for rents, incomes, etc. can be expected to have been influenced by this. Any subsequent changes in multiple starts due to movements in market variables are referred to as induced, or indirect, impacts of the program. These are estimated in Appendix D.)

It will be readily observed from the figures appearing in the Tables that the net direct impacts are always positive while a program is in effect and continue to be positive for several years after a program's expiry. The explanation for the former is that positive coefficients are associated with the capitalized value of each program's subsidies in Regressions (1a)-(3a). The explanation for the positive post-expiry net impacts is that each of these Regressions contains a lagged dependent variable with a coefficient of approximately 0.5.

The positive effects of lagged starts in each Regression is attributable to the presence of adjustment costs. It is apparently costly for builders to make large quarter-to-quarter changes in the levels of starts. Consequently, a subsidy program that provides builders with an incentive to increase starts today will also result in higher starts tomorrow, even if the subsidy program has expired.

Included in the last column of each of Tables 8-11 are values obtained from CMHC for the number of multiple housing units started under each of the subsidy programs. These are referred to in the tables as the

gross direct impacts for the respective subsidy programs. For example, the fourth column of Table 8 reports that a total of 57,878 multiple units that were started between 1970 and 1987 received subsidies under the Limited Dividend program.

Comparison of the net direct impact estimates derived from Regressions (1a)-(3a) with the gross direct impacts provides a range of estimates for the direct offsets on multiple starts for each of the programs. The figures of most interest here are the net direct impacts and offsets derived from the preferred Regression (3a). Estimates derived from Regressions(1a) and (2a) serve to provide a range of estimates for the true values of the net direct impacts.

The Regression (3a) simulations yield the smallest estimates for the net direct impacts for all of the subsidy programs, except MURB's, for which the Regression (1a) simulations provide smaller estimates. Nonetheless, the net impact estimates derived from Regression (3a) are not small: They range from between 20.55 percent of the gross direct impacts (for the ARP program) to 46.6 percent of the gross direct impacts (for the LD program). The evidence provided here is that each of the subsidy programs made an economically significant and positive contribution to multiple housing starts over the 1970-87 time period.

Table 12 provides some alternative information concerning the sensitivity of the net direct impacts to values for the key coefficients appearing in the preferred regression equation. The "High" estimates shown in this table were derived by re-performing the single-equation dynamic simulations described above, after increasing the coefficients on both the MURB and (LD+ARP+CRSP) in Regression (3a) by one standard

deviation, respectively. The "Low" estimates in Table 12 were derived from single-equation dynamic simulations of Regression (3a), after having reduced the coefficient on the MURB variable by one standard deviation and setting the coefficient on the (LD+ARP+CRSP) variable to zero. (A one standard deviation reduction in the coefficient of the later variable would have produced a negative coefficient, but a value of zero here represents a more intuitively-plausible lower limit.

The standard errors of the estimated coefficients are such that the "High" estimates for each of the LD, ARP and CRSP programs are approximately 2.5 times the point estimates in the third columns of Tables 8, 10 and 11, respectively. The "High" and "Low" estimates for the MURB program are roughly 1.5 times and 0.5 times, respectively, the point estimates from the third column of Table 9.

6. Variable Listing and Definitions

ARP	the deflated value of the capitalized value of subsidies associated with the Assisted Rental Program; computed as $(PV11-PV1)/CPI$ or $(PV12-PV2)/CPI$ when used in regressions involving $(PV1/CPI)$ or $(PV2/CPI)$, respectively.	
CPI	the all-items Consumer Price Index, = 100.0 in 1981.	
CRSP	the deflated value of the capitalized value of subsidies associated with the Canada Rental Subsidy Program; computed as $(PV7-PV1)/CPI$ or $(PV7-PV2)/CPI$ when used in regressions involving $(PV1/CPI)$ or $(PV2/CPI)$, respectively.	
D1	dummy variable	$\begin{cases} - 0 & \text{prior to 1974:4.} \\ - 0.24 & \text{for 1974:4 to 1975:2.} \\ - 1.0 & \text{for 1975:3 to 1984:2.} \\ - 0.76 & \text{for 1984:3 to 1987:4.} \end{cases}$
$\overline{D1}$	$1/4(D1 + D1_{-1} + D1_{-2} + D1_{-3})$.	
D2	dummy variable	$\begin{cases} - (\ln CPI - \ln CPI_{-4}) & \text{prior to 1975:3.} \\ - 0.08 & \text{for 1975:3 to 1977:3.} \\ - 0.06 & \text{for 1977:4 to 1985:3.} \\ - 0.04 & \text{for 1985:3 to 1986:4.} \\ - 0.052 & \text{for 1987:1 to 1987:4.} \end{cases}$
DD2	dummy variable	$\begin{cases} - 1 & \text{in 1970:1, 1974:4, 1975:2,} \\ & \text{1976:2, 1978:4, 1980:4,} \\ & \text{1982:1.} \\ - 0 & \text{elsewhere.} \end{cases}$
D3	dummy variable	$\begin{cases} - 0 & \text{prior to 1982:4.} \\ - 1 & \text{from 1982:4 to 1987:4.} \end{cases}$

D4	dummy variable	$\begin{cases} - 0 & \text{prior to 1978:4.} \\ - 1 & \text{from 1978:4 to 1987:4.} \end{cases}$
DCHOSP	dummy variable	$\begin{cases} - 1 & \text{for 1982:3 to 1983:2.} \\ - 0 & \text{elsewhere.} \end{cases}$
DMURB	dummy variable	$\begin{cases} - 1 & \text{for 1974:4 to 1979:4.} \\ - 0 & \text{and 1980:4 to 1981:4.} \\ & \text{elsewhere.} \end{cases}$
HSM	multiple-unit housing starts, seasonally-adjusted quarterly totals.	
HSS	single-detached housing starts, seasonally-adjusted quarterly totals.	
i	nominal rate of interest in conventional five-year mortgages, quarterly average, expressed as a per cent.	
K_M	end-of-quarter stock of multiple houses, in thousands of units; interpolated quarterly from end-of-year figures using quarterly completions.	
K_S	end-of quarter stock of single-detached houses, in thousands of units; interpolated quarterly for end-of-year figures using quarterly completions.	
LD	the deflated value of the capitalized value of subsidies associated with the Limited Dividend Program; computed as $(PV3-PV1)/CPI$ or $(PV3-PV2)/CPI$ when used in regressions involving $(PV1/CPI)$ or $(PV2/CPI)$, respectively.	
MURB	computed as $(PV2-PV1)/CPI$ when used in regressions involving $(PV1/CPI)$; equal to DMURB when used in regression involving $(PV2/CPI)$.	
N	Canadian resident population during last month in quarter, in thousands.	
O	occupancy rate of apartment buildings; computed as $(1.0 - v/100)$.	

P_S	index of the selling prices of newly-completed single-detached homes, including land, = 100.0 in 1981.
PV1	nominal capitalized value of a prototypical non-subsidized apartment building, in dollars per unit. (See Appendix 1.)
PV1R	(PV1/CPI).
PV2	nominal capitalized value of a prototypical apartment building with a M.U.R.B. certificate in dollars per unit. (See Appendix 1.)
PV3	nominal capitalized value of a prototypical apartment building with participation in CMHC's Limited Dividend program, in dollars per unit. (See Appendix 1.)
PV7	nominal capitalized value of a prototypical apartment building with participation in the Canada Rental Assistance Program, in dollars per unit. (See Appendix 1.)
PV11	nominal capitalized value of a prototypical apartment building with participation in CMHC's Assisted Rental Program and with no M.U.R.B. certificate, in dollars per unit. (See Appendix 1.)
PV12	nominal capitalized value of a prototypical apartment building with participation in A.R.P. and with a M.U.R.B. certificate, in dollars per unit. (See Appendix 1.)
r	<u>ex post</u> real rate of interest, measured as a per cent; computed as $i - 400*(CPI_{+1}/CPI - 1.0)$.
R	the rent component of the Consumer Price Index, = 100.0 in 1981.
S	<u>ex post</u> user cost of capital for a single-detached house, computed as $P_S(1 + (i/400) + \delta_S) - P_{S+1}$
S01, S02, S03, S04	computed values for S with values δ_S equal to 0.01, 0.02, 0.03, 0.04, respectively.
v	vacancy rate for apartment buildings, measured as a per cent; interpolated quarterly on a straight-line basis from values from CMHC's semi-annual survey of vacancies.
\bar{v}	$1/4(v + v_{-1} + v_{-2} + v_{-3})$
Y	personal disposable income deflated by the CPI, seasonally adjusted at annual rates, in millions of dollars at 1981 prices.

YP permanent income, computed as $YP = 0.4*(0.6 YP_{-1} + Y)$, with
a benchmark.value equal to Y in 1961:1.

References

- G. Carliner, "Income Elasticity of Housing Demand," Review of Economics and Statistics, 55 (Nov. 1973) 528-532.
- S.J. Grossman, A. Melino and R. Shiller, "Estimating the Continuous Time Consumption Based Asset Pricing Model," Journal of Business and Economic Statistics, 5 (1987) 315-327.
- J.R. Kearl, "Inflation, Mortgages, and Housing," Journal of Political Economy, 87 (Oct. 1979) 1115-1138.
- R. Klemkosky and D. Lasser, "An Efficiency Analysis of the T-Bond Futures Market," Journal of Futures Markets (1985).
- W.A. Leigh, "Economic Depreciation of the Residential Housing Stock of the United States, 1950-1970," Review of Economics and Statistics, 62 (1980) 200-206.
- E.H. Oksanen, "Housing Demand in Canada, 1947-1962: Some Preliminary Experimentation," Canadian Journal of Economics and Political Science, 32 (Aug. 1966) 302-318.
- J.M. Poterba, "Tax Subsidies to Owner-Occupied Housing; An Asset-Market Approach," Quarterly Journal of Economics, 94 (Nov. 1984) 729-752.
- L.B. Smith, "A Model of the Canadian Housing and Mortgage Markets," Journal of Political Economy, 77 (Sept./Oct. 1969) 795-816.
- G.R. Sparks, "The Effect of CHOSP on Housing Starts," Canadian Mortgage and Housing Corporation (1986) 18 pages (mimeo).
- R. Topel and S. Rosen, "Housing Investment in the United States," Journal of Political Economy, 96 (1988) 718-740.

Table 1

Regression Results for the User Cost of Single-Detached Houses

Sample Period: 1970:1 to 1987:4

Dependent Variables	Independent Variables					$\hat{\rho}$	Standard Error of Regression	D.W.	\bar{R}^2	F(p,q)
	Intercept	Y/N	R/CPI	K/N	(K/N) ₋₁					
1. S01	1.0167 (3.94)	-1.6624 (-0.94)	-0.2992 (-3.92)	-31.9534 (-2.27)	28.5255 (1.98)	---	0.0210	1.03	0.37	F(4,67)=11.6
2. S02	1.0002 (3.82)	-1.5286 (-0.85)	-0.2889 (-3.72)	-32.1250 (-2.25)	28.7143 (1.96)	---	0.0213	1.01	0.36	F(4,67)=10.8
3. S03	0.9837 (3.69)	-1.3947 (-0.76)	-0.2786 (-3.53)	-32.2970 (-2.22)	28.9031 (1.94)	---	0.0217	0.98	0.34	F(4,67)=9.99
4. S04	0.9672 (3.55)	-1.2609 (-0.67)	-0.2684 (-3.32)	-32.4688 (-2.18)	29.0919 (1.91)	---	0.0222	0.94	0.31	F(4,67)=9.15
5. S04	1.1283 (4.27)	-0.7891 (-0.42)	-0.3443 (-4.80)	-4.3079 (-1.97)	---	---	0.0226	0.86	0.29	F(3,68)=10.57
6.* S04	1.2279 (2.97)	-1.1153 (-0.58)	-0.3791 (-3.2335)	-4.5194 (-1.73)	---	0.0918 (5.73)	0.0186	1.85	0.64	F(4,67)=20.01
7. S04	-0.0225 (-0.18)	0.4677 (0.22)	---	0.2189 (2.26)	---	---	0.0256	0.69	0.06	F(2,69)=3.28
8.* S04	-0.0517 (-0.39)	-0.4358 (-0.22)	---	1.0032 (2.13)	---	0.0896 (7.22)	0.0196	1.87	0.46	F(3,68)=21.48

Figures in parentheses are t-statistics.

* Equation estimated with correction for first-order serial correlation of the residuals.

Table 2

Regression Results for the Price of Single-Detached Houses

Sample Period: 1970:1 to 1987:4

Dependent Variable: $\ln(P_S/CPI)$

Mean (Standard Deviation) of Dependent Variable: -0.0028 (0.1457)

	Independent Variables						$\hat{\rho}$	Standard Error of Regression	D.W.	\bar{R}^2	F(p,q)
	Intercept	$\ln(Y/CPI)$	$\ln(R/CPI)$	$\ln(K_S/N)$	$rx10^2$	DCHOSP					
1.	-4.4264 (-1.36)	1.1525 (3.35)	-0.8722 (-2.10)	-4.4380 (-7.67)	-0.0153 (-4.29)	-0.0359 (-0.85)	---	0.0751	0.36	0.73	F(5,66)=40.32
2.	-3.8787 (-1.39)	1.2858 (4.20)	-0.6687 (-1.97)	-4.2812 (-7.82)	-0.0152 (-4.28)	---	---	0.0749	0.38	0.74	F(4,67)=50.42
3.*	-0.9310 (-0.50)	0.1208 (0.69)	-0.5484 (-1.71)	-0.7587 (-0.87)	-0.0019 (-1.77)	---	0.9811 (68.71)	0.0217	0.78	0.98	F(5,66)=626.9
4.	-1.9956 (-1.57)	1.7254 (8.01)	---	-3.7254 (-7.75)	-0.0125 (-3.69)	0.0149 (0.42)	---	0.0770	0.36	0.72	F(4,67)=46.90
5.	-1.9337 (-1.53)	1.7256 (8.06)	---	-3.7195 (-7.79)	-0.0122 (-3.721)	---	---	0.0765	0.34	0.72	F(3,68)=63.24
6.*	0.6955 (0.48)	0.1857 (1.07)	---	0.1867 (0.27)	-0.0014 (-1.34)	---	0.9821 (70.92)	0.0220	0.77	0.98	F(4,67)=761.8

Figures in parentheses are t-statistics.

*Equation estimated with correction for first-order serial correlation of the residuals.

Table 3

Regression Results for Single-Detached Housing Starts

Sample Period: 1970:1 to 1987:4

Dependent Variable: (HSS/N) (in units of quarterly starts (S.A.) per 1,000 population.)

Mean (Standard Deviation) of Dependent Variable: 1.0689 (0.3957).

	Independent Variables						$\hat{\rho}$	Standard Error of Regression	D.W.	\bar{R}^2	F(p,q)
	Intercept	P _G /CPI	r	i	DCHOSP	(HSS/N) ₋₁					
1.	0.7324 (1.59)	0.5177 (1.27)	-3.1212 (-1.92)	—	-0.1470 (-0.91)	—	—	0.3531	0.57	0.24	F(3,68)=7.05
2.*	-0.2504 (-0.31)	1.3307 (1.69)	-0.1255 (-0.10)	—	0.2456 (1.40)	—	0.7833 (11.11)	0.2317	2.10	0.66	F(4,67)=35.02
3.	0.1805 (0.56)	1.0734 (0.38)	-0.3966 (0.34)	—	-0.8331 (-0.76)	0.7539 (8.94)	—	0.2901	2.12	0.63	F(4,67)=31.46
4.	1.0264 (3.20)	1.0573 (4.03)	—	-8.3358 (-5.20)	0.1595 (1.03)	—	—	0.3067	0.68	0.40	F(3,68)=16.73
5.*	0.4829 (0.75)	1.2411 (2.20)	—	-5.4114 (-2.13)	0.2321 (1.38)	—	0.6918 (8.15)	0.2278	1.98	0.67	F(4,67)=36.81
6.	0.4479 (1.74)	0.2897 (1.28)	—	-3.0896 (-2.18)	0.0337 (0.28)	0.6585 (7.18)	—	0.2322	1.99	0.66	F(4,67)=34.79
7.*	0.5674 (0.92)	1.1307 (2.11)	—	-5.0638 (-2.07)	—	—	0.6723 (7.77)	0.2293	2.02	0.66	F(3,68)=47.81
8.	0.4383 (1.74)	0.2736 (1.26)	—	-2.8910 (-2.36)	—	0.6624 (7.36)	—	0.2306	2.01	0.66	F(3,68)=46.99

Figures in parentheses are t-statistics.

*Equation estimated with correction for first-order serial correlation of the residuals.

Table 4

Parameter Estimates for the Demand for Occupied Multiple Housing Units

Sample Period: 1970:1 to 1987:4

$$\text{Estimated Equation: } \Delta \ln(0_t K_{M_t} / N_t) = \gamma_{M_0} (1 - a_4 D1_t) [a_0 + a_1 \ln(Y_t / N_t) + a_2 \ln(R_t / CPL_t) - \ln(0_{t-1} K_{M_{t-1}} / N_{t-1})]$$

Mean (Standard Deviation) of Dependent Variable: 0.0044 (0.0052)

Method of Estimation: Nonlinear Least Squares

Regression Number	Parameter					Standard Error of Regression x10 ²	*R ²	*F(p,q)
	a ₀	a ₁	a ₂	γ _{M₀}	a ₄			
1.	1.5722 (4.69)	0.2316 (4.23)	0.0336 (1.35)	0.2954 (3.04)	0.7231 (7.04)	0.3660	0.73	F(5,67)=49.49
2.	2.0927 (3.35)	0.2883 ⁺ (3.08)	0.0929 (3.25)	0.2979 (2.24)	0.7911 (9.21)	0.3864	0.70	F(5,67)=44.19
3.	3.3249 (13.08)	0.5676 (20.40)	-0.1274 (-3.95)	1.0 -	0.9473 (21.61)	0.4388	0.60	F(4,68)=42.78
4.	5.0415 (15.22)	0.7539 ⁺ (20.84)	0.0834 (2.5)	1.0 -	0.9163 (15.97)	0.4451	0.59	F(4,68)=41.57

Figures in parenthesis are t-statistics.

+Parameter estimated using permanent income.

*R² and F-statistics for equations estimated with non-linear least squares take as the alternative to the null hypothesis the hypothesis H1: Dependent Variable = 0.

Table 5

Parameter Estimates for the Rate of Change in Rents

Sample Period: 1970:2 to 1987:4

Estimated Equation: $(\ln R_t - \ln R_{t-4}) = (1 - \lambda_0 \bar{D1}_t) [\lambda_1 (\ln CPI_t - \ln CPI_{t-4})$
 $+ \lambda_2 (\bar{v}_t - v^*)] + \lambda_0 \bar{D1}_t D2_t$

Mean (Standard Deviation) of Dependent Variable: 0.0451 (0.0207)

Method of Estimation: Nonlinear Least Squares

Regression Number	*Parameter				Standard Error of Regres- sion	*R ²	*F(p,q)
	λ_1	λ_2	v^*	λ_0			
1.	-0.0826 (-0.73)	-0.0099 (-2.87)	5.8938 (8.66)	0.8078 (12.62)	0.0115	0.95	F(4,68)=334.2
2.	1.0 -	0.0211 (5.81)	5.2150 (14.15)	1.0029 (23.72)	0.0162	0.90	F(4,68)=168.3
3.	1.0 -	0.0027 (0.80)	14.5716 (0.98)	0.47 -	0.0217	0.82	F(4,68)=93.67
4.	1.0 -	-0.0074 (-1.39)	2.5 -	0.9541 (6.51)	0.0297	0.66	F(4,68)=49.89
5.	1.0 -	-0.0035 (-0.74)	2.5 -	0.47 -	0.0316	0.62	F(4,68)=44.04
6.	1.0 -	-0.02 -	1.1569 (4.27)	0.7658 (5.05)	0.0272	0.72	F(4,68)=59.53

Figures in parenthesis are of statistics.

*R² and F-statistics for equations estimated with non-linear least squares take as the alternative to the null hypothesis the hypothesis H1: Dependent Variable = 0.

Table 6

Regression Results for Multiple Starts Using PVIR As A Proxy for Market Price

Sample Period: 1970:1 to 1987:4.

Dependent Variable: $(HSM/N) \times 10^3$ (expressed as quarterly starts (S.A.) per/million population).

Mean (Standard Deviation) of Dependent Variable: 1068.86 (395.71).

Inter- cept	PVIR	PVIR*i	PVIR*D4	i	D1	D3	D4	D3*i	MURB	LD	CRSP	ARP	$(HSM/N)_{-1} \times 10^3$	$\hat{\rho}$	S.E.R.	\bar{R}^2	F(p,q)
1. 2972.06 (6.18)	-2.4467 (-2.94)	-	-	-71.76 (-2.43)	76.76 (0.39)	-	-430.58 (-2.77)	-1.5570 (-0.15)	-1.6069 (-0.93)	-6.5019 (-2.60)	-6.7401 (-1.07)	3.9739 (1.37)	-	-	219.4	0.69	F(9,62)=18.77
2. 2262.13 (4.43)	-2.1371 (-2.71)	-	-	-67.13 (-2.42)	48.69 (0.26)	-	-210.51 (-2.71)	3.2593 (0.33)	-0.5711 (-0.34)	-3.6673 (-1.45)	-3.5250 (-0.59)	4.0456 (1.49)	0.3611 (3.01)	-	206.4	0.72	F(10,61)=20.00
3. 3252.26 (6.82)	-2.7440 (-3.33)	-	-	-100.95 (-3.85)	369.63 (2.84)	-	-520.24 (-3.86)	-2.4118 (-0.27)	-0.8095 (-0.52)	+	1.8284 (-1.42)	+	-	-	225.8	0.67	F(7,64)=22.01
4* 3173.84 (5.94)	-2.5224 (-2.89)	-	-	-104.45 (-3.34)	295.44 (1.97)	-	-367.57 (-2.30)	-6.9958 (-0.67)	-0.4815 (-0.28)	+	-0.4123 (-0.27)	+	-	0.3679 (3.15)	212.6	0.71	F(8,63)=22.87
5. 2356.68 (4.62)	-2.3103 (-2.99)	-	-	-86.22 (-3.51)	251.77 (2.06)	-	-246.48 (-1.67)	3.6426 (0.44)	0.0794 (0.05)	+	-0.0041 (-0.00)	+	0.4102 (3.47)	-	208.5	0.72	F(8,63)=24.09
6. 1693.15 (4.15)	-1.0966 (-1.7)	-	-0.2899 (-0.82)	-63.91 (-3.27)	-	-	-	-0.1913 (-0.02)	1.4837 (1.14)	+	0.8466 (0.72)	+	0.4715 (4.08)	-	213.2	0.71	F(7,64)=25.80
7. 1345.41 (8.68)	4.7000 (7.56)	-0.4615 (-8.30)	-0.7580 (-3.42)	-	-	-	-	-	-0.7189 (-0.57)	+	1.8459 (1.72)	+	-	-	231.5	0.66	F(5,66)=28.29
8* 1319.27 (6.79)	4.3157 (4.90)	-0.4246 (-5.91)	-0.6881 (-2.37)	-	-	-	-	-	-1.0867 (-0.71)	+	2.2049 (1.60)	+	-	0.4176 (3.68)	211.9	0.71	F(6,65)=30.43
9. 812.15 (4.33)	2.7883 (3.88)	-0.2991 (-4.75)	-0.2209 (-0.93)	-	-	-	-	-	-0.0798 (-0.07)	+	1.9242 (2.00)	+	0.4382 (4.21)	-	206.5	0.72	F(6,65)=32.79
10. 892.29 (3.99)	2.3431 (2.77)	-0.2657 (-3.59)	-	-	-	-46.63 (-0.45)	-82.09 (-0.66)	-	0.0921 (0.08)	+	1.4038 (1.11)	+	0.4163 (3.65)	-	206.6	0.72	F(7,64)=28.07
11. 904.53 (4.10)	2.1770 (2.86)	-0.2602 (-3.58)	-	-	-	-	-105.10 (-0.93)	-	0.2489 (0.22)	+	1.3589 (1.09)	+	0.4321 (4.00)	-	205.3	0.73	F(6,65)=33.11

Figures in parentheses are t-statistics.

*Corrected for first-order serial correlation in the residuals.

PVIR = PV1/CPI.

Table 7

Alternative Regression Equations
Determining Multiple Starts

Sample Period: 1970:1 to 1987:4.

Dependent Variable: $(HSM/N) \times 10^3$ (expressed as quarterly starts (S.A.) per 1 million population).

Mean (Standard Deviation) of Dependent Variable: 1068.86 (395.71)

"Price" Variable	Price	Price <i>i</i>	Intercept	D4	MURB	(LD+ARP+CRSP)	$(HSM/N)_{-1} \times 10^3$	Standard Error of Regression	D.W.	\bar{R}^2	F(p,q)
1. PV1/CPI	2.1770 (2.86)	-0.2602 (-3.58)	904.528 (4.10)	-105.096 (-0.93)	0.2489 (0.22)	1.3588 (1.09)	0.4321 (4.00)	205.34	1.98	0.73	F(6,65)=33.11
2. PV2/CPI	1.8237 (2.74)	-0.2798 (-3.79)	1056.623 (4.70)	-44.025 (-0.39)	109.184 (1.88)	2.2452 (1.48)	0.4169 (3.98)	203.61	1.90	0.74	F(6,65)=33.86
3. R/CPI	694.599 (2.16)	-24.8887 (-1.59)	27.6213 (0.06)	-50.7258 (-0.32)	2.5534 (1.97)	0.7482 (0.57)	0.5029 (4.50)	216.82	2.07	0.70	F(6,65)=28.58
<u>Reduced form equation</u>											
Y/N	K_M / N_{-1}	i	Intercept	D4	MURB	(LD+ARP+CRSP)	$(HSM/N)_{-1}$				
4. 28.2534 (1.19)	-0.0305 (-1.38)	-25.5011 (-1.60)	2505.72 (2.46)	-152.639 (-0.96)	2.0697 (1.56)	0.9078 (0.66)	0.4913 (4.27)	218.44	2.02	0.69	F(6,65)=28.00

Figures in parenthesis are t-statistics.

*Variable denoted by MURB is $(PV2-PV1)/CPI$ in Regressions 1, 3, 4, and is DMURB in Regression 2.

Variable denoted by (LD+ARP+CRSP) is $((PV3-PV1)+(PV11-PV1)+(PV7-PV1))/CPI$ in Regressions 1, 3, 4 and is $((PV3-PV2)+(PV12-PV2)+(PV7-PV2))/CPI$ in Regression 2.

Table 7a

Alternative Regression Equations
Determining Multiple Starts

Sample Period: 1970:1 to 1987:4.

Dependent Variable: $(HSM/N) \times 10^3$ (expressed as quarterly starts (S.A.) per 1 million population).

Mean (Standard Deviation) of Dependent Variable: 1068.86 (395.71)

"Price" Variable	Price	Price*1	Intercept	D4	MURB	(LD+ARP+CRSP)	$(HSM/N)_{-1} \times 10^3$	DD2	Standard Error of Regression	D.W.	\bar{R}^2	F(p,q)
1. PV1/CPI	2.1770 (2.86)	-0.2602 (-3.58)	904.528 (4.10)	-105.096 (-0.93)	0.2489 (0.22)	1.3588 (1.09)	0.4321 (4.00)	---	205.34	1.98	0.73	F(6,65)=33.11
1a. PV1/CPI	2.0152 (2.67)	-0.2508 (-3.49)	936.226 (4.29)	-119.329 (-1.07)	0.3732 (0.34)	1.5773 (1.28)	0.4318 (4.06)	-145.653 (-1.73)	202.27	2.08	0.74	F(7,64)=29.56
2. PV2/CPI	1.8237 (2.74)	-0.2798 (-3.79)	1056.623 (4.70)	-44.025 (-0.39)	109.184 (1.88)	2.2452 (1.48)	0.4169 (3.98)	---	203.61	1.90	0.74	F(6,65)=33.86
2a. PV2/CPI	1.6939 (2.61)	-0.2812 (-3.93)	1135.626 (5.13)	-60.501 (-0.56)	136.990 (2.36)	2.3961 (1.62)	0.4014 (3.93)	-183.315 (-2.20)	197.83	2.01	0.75	F(7,64)=31.30
3. R/CPI	694.599 (2.16)	-24.8887 (-1.59)	27.6213 (0.06)	-50.7258 (-0.32)	2.5534 (1.97)	0.7482 (0.57)	0.5029 (4.50)	---	216.82	2.07	0.70	F(6,65)=28.58
3a. R/CPI	636.921 (1.98)	-20.6731 (-1.31)	60.7786 (0.14)	-77.1597 (-0.49)	2.5413 (1.98)	0.8851 (0.67)	0.5026 (4.53)	-130.090 (-1.44)	215.04	2.16	0.70	F(7,64)=25.20

Figures in parenthesis are t-statistics.

*Variable denoted by MURB is $(PV2-PV1)/CPI$ in Regressions 1, 1a, 3, and 3a is $UMURB$ in Regressions 2 and 2a.Variable denoted by $(LD+ARP+CRSP)$ is $((PV3-PV1)+(PV11-PV1)+(PV7-PV1))/CPI$ in Regressions 1, 1a, 3 and 3a is $((PV3-PV2)+(PV12-PV2)+(PV7-PV2))/CPI$ in Regressions 2 and 2a.

Table 8

Estimated Net Direct Impacts on Multiple Starts of
the Limited Dividend Program

(number of starts)

Year	*Estimated Net Direct Impacts			†Gross Direct Impacts
	Regression 1a	Regression 2a	Regression 3a	
1970	5,253	7,756	3,153	19,609
1971	3,928	5,596	2,587	11,507
1972	3,868	5,589	2,472	8,797
1973	3,915	5,655	2,500	4,526
1974	8,624	10,840	5,302	2,544
1975	13,331	7,756	8,361	10,895
1976	3,025	1,715	2,439	-
1977	106	45	158	-
1978	4	1	10	-
1979	-	-	-	-
1980	-	-	-	-
1981	-	-	-	-
1982	-	-	-	-
1983	-	-	-	-
1984	-	-	-	-
1985	-	-	-	-
1986	-	-	-	-
1987	-	-	-	-
TOTAL	42,057	44,953	26,983	57,878
TOTAL AS FRACTION OF GROSS IMPACT	0.727	0.777	0.466	1.000

*Estimates derived from dynamic simulations of the regression equations 1a, 2a and 3a, reported in Table 7a.

†Total number of starts receiving subsidies under the program.

Table 9

Estimated Net Direct Impacts on Multiple Starts of
the M.U.R.B. Program

(number of starts)

Year	*Estimated Net Direct Impacts			†Gross Direct Impacts
	Regression 1a	Regression 2a	Regression 3a	
1970	-	-	-	-
1971	-	-	-	-
1972	-	-	-	-
1973	-	-	-	-
1974	280	3,081	1,908	-
1975	1,875	19,412	13,909	8,517
1976	2,257	21,005	17,434	35,219
1977	2,060	21,291	16,118	82,265
1978	2,143	21,515	16,607	80,089
1979	1,342	21,730	10,798	76,550
1980	792	6,889	6,306	-
1981	5,074	20,905	37,382	61,500
1982	1,088	3,629	10,743	-
1983	38	95	692	-
1984	1	3	45	-
1985	-	-	3	-
1986	-	-	-	-
1987	-	-	-	-
TOTAL	16,950	139,555	131,955	344,140
TOTAL + TOTAL GROSS DIRECT IMPACT	0.049	0.406	0.383	1.000

*Estimates derived from dynamic simulations of the regression equations 1a, 2a and 3a, reported in Table 7a.

†Total number of starts receiving subsidies under the program.

Table 10

Estimated Net Direct Impacts on Multiple Starts of
the Assisted Rental Program (ARP)

(number of starts)

Year	*Estimated Net Direct Impacts			†Gross Direct Impacts
	Regression 1a	Regression 2a	Regression 3a	
1970	-	-	-	-
1971	-	-	-	-
1972	-	-	-	-
1973	-	-	-	-
1974	-	-	-	-
1975	10,175	15,723	6,008	22,351
1976	13,449	20,370	8,651	25,151
1977	8,771	13,600	5,779	57,053
1978	5,887	9,504	3,894	18,198
1979	963	1,420	811	-
1980	34	37	52	-
1981	1	1	3	3
1982	-	-	-	-
1983	-	-	-	-
1984	-	-	-	-
1985	-	-	-	-
1986	-	-	-	-
1987	-	-	-	-
TOTAL	39,280	60,655	25,198	122,753
TOTAL + TOTAL GROSS DIRECT IMPACT	0.320	0.494	0.205	1.000

*Estimates derived from dynamic simulations of the regression equations 1a, 2a and 3a, reported in Table 7a.

†Total number of starts receiving subsidies under the program.

Table 11

Estimated Net Direct Impacts on Multiple Starts of
the Canada Assisted Rental Program (CRSP)

(number of starts)

Year	*Estimated Net Direct Impacts			†Gross Direct Impacts
	Regression 1a	Regression 2a	Regression 3a	
1970	-	-	-	-
1971	-	-	-	-
1972	-	-	-	-
1973	-	-	-	-
1974	-	-	-	-
1975	-	-	-	-
1976	-	-	-	-
1977	-	-	-	-
1978	-	-	-	-
1979	-	-	-	-
1980	-	-	-	-
1981	-	-	-	-
1982	3,566	5,274	2,132	10,405
1983	5,258	7,622	3,314	10,265
1984	5,606	8,088	3,584	3,452
1985	2,327	3,210	1,657	-
1986	81	84	107	-
1987	4	3	9	-
TOTAL	16,842	24,281	10,803	24,122
TOTAL	0.698	1.007	0.448	1.000
+ TOTAL GROSS IMPACT				

*Estimates derived from dynamic simulations of the regression equations 1a, 2a and 3a, reported in Table 7a.

†Total number of starts receiving subsidies under the program.

Table 12

"High" and "Low" Estimates of the Net Direct Impacts of
the Subsidy Programs on Multiple Starts

(in numbers of starts)

*Estimated Net Impacts

Year	<u>LD Program</u>		<u>MURB Program</u>		<u>ARP Program</u>		<u>CRSP Program</u>	
	"High"	"Low"	"High"	"Low"	"High"	"Low"	"High"	"Low"
1970	7,883	0	-	-	-	-	-	-
1971	6,468	0	-	-	-	-	-	-
1972	6,180	0	-	-	-	-	-	-
1973	6,250	0	-	-	-	-	-	-
1974	13,255	0	2,862	954	-	-	-	-
1975	21,578	0	20,864	6,954	15,020	0	-	-
1976	6,098	0	26,151	8,717	21,628	0	-	-
1977	395	0	24,177	8,059	14,448	0	-	-
1978	25	0	24,910	8,304	9,735	0	-	-
1979	3	-	16,197	5,372	2,028	0	-	-
1980	-	-	9,459	3,153	1,300	0	-	-
1981	-	-	58,073	18,691	8	0	-	-
1982	-	-	16,114	5,372	-	-	5,330	0
1983	-	-	1,038	346	-	-	8,285	0
1984	-	-	68	22	-	-	8,960	0
1985	-	-	5	2	-	-	4,142	0
1986	-	-	-	-	-	-	268	0
1987	-	-	-	-	-	-	22	0

*Estimates derived from single-equation simulations of Regression (3a) with program coefficients raised ("High") or lowered ("Low") by one standard deviation.

May 16, 1989

Appendix D

SIMULATIONS OF THE MACROECONOMIC IMPACTS OF THE CMHC RENTAL SUBSIDY PROGRAMS ON THE CANADIAN ECONOMY: 1971-1987

Institute for Policy Analysis

University of Toronto

May 16, 1989

APPENDIX D

Simulations of the Macroeconomic Impacts of the CMHC Rental Subsidy Programs on the Canadian Economy: 1971-1987

This Appendix reports the results of counter-factual simulation experiments designed to estimate the net impacts of the various CMHC rental subsidy programs on the Canadian macro-economy.

The simulations were conducted using an appropriately-modified version of the FOCUS econometric model of the Canadian economy. The FOCUS model is a large scale, quarterly, macro-econometric model developed and maintained by the Institute for Policy Analysis at the University of Toronto.

The modifications made to the FOCUS model for purposes of this exercise consisted of incorporating into the structure of FOCUS the sub-model of the Canadian residential housing market developed in this study and described in detail in Appendix C. The sub-model regression equations, designated as the "preferred" results in Appendix C, were used to replace the corresponding equations in FOCUS. Specifically, Appendix C's regression equations 5 (Table 2), 5* (Table 3), 3 (Table 4), 4 (Table 5) and 3a (Table 7a) were used, respectively, to determine single-family

housing prices, single starts, occupancy/vacancy rates, rents and multiple starts in the simulation exercises. The remainder of the several hundred equations in the FOCUS model were left unaltered.

The simulation exercises consisted of a four-step procedure, each of which is detailed below. By way of overview, the basic idea was to utilize the modified FOCUS model to first re-create the actual historical experience of the Canadian economy over the 68 quarter time interval starting in 1971:1 and ending in 1987:4. (This interval was chosen because data limitations underlying certain of the equations in the FOCUS model make simulation prior to 1971:1 extremely difficult.) The model was then used to generate a series of alternative, or counter-factual, solutions over this same time interval. Each alternative presents a scenario of how the Canadian might have performed had a hypothetical event, or series of events, occurred.

In the alternative solutions examined here the hypothetical events deal with CMHC's rental subsidy programs. One alternative solution depicts how the Canadian macro-economy might have performed had the Limited Dividend Program been terminated at the start of 1971. Another depicts how macro-economic performance might have looked had the MURB program never been initiated. An alternative solution was performed for each of the four rental subsidy programs in isolation. The combined effects of "shutting down" all four of the programs, collectively, yielded a fifth alternative.

Estimates of the impacts of the programs, individually and collectively, on macroeconomic performance may be computed by comparing actual economic performance over the 1971:1 to 1987:4 simulation interval

with each of the alternative solutions. For example, the actual value for real GDP in 1978 was \$325,751 million (in 1981 prices). The alternative solution based on shutting down the MURB program produced an estimate for 1978 real GDP of \$325,319 million, a value that is lower by \$432 million. Since the only difference between the model specifications which underlie the solution that replicates history and this alternative solution is that MURB's are "on" in the former and "off" in the latter, the \$432 million difference in estimated real GDP may be deemed to be the impact of MURB's on the 1978 value for real GDP. In other words, the simulations reveal that real GDP was \$432 million higher in 1978 than it might otherwise have been had the MURB program never been initiated.

Impact estimates for MURB's calculated in similar fashion for a selected number of macroeconomic variables, are reported in Table 3. Impact estimates for Limited Dividends, ARP, CRSP and all programs combined are reported in Tables 2, 4, 5, and 6, respectively.

A. Details of Simulation Mechanics

Before discussing the impact estimates, it is useful to review in some detail the actual mechanics that give rise to the simulations results.

1. The Control Solution

The term "control solution" is used to describe the model solution against which the various alternative solutions are compared in order to compute the impact estimates. In this case our control solution is a dynamic solution of the FOCUS model that exactly replicates actual

economic events in every quarter from 1971:1 to 1987:4.

Since no model, no matter how good, will perfectly replicate actual historical data on its own, some "fine tuning" of the FOCUS model was required to generate the control solution. How this was done is rather easily illustrated using a "typical" equation from the model:

$$Y_{1t} = a_0 + a_1 Y_{2t} + a_2 Y_{1t-1} + a_3 x_t (+R_{1t}).$$

This equation determines endogenous variable Y_1 in time period t as a combination of parameters (the a_i 's) and three independent variables. (Ignore the R_{1t} term for now.) The independent variables consist of the contemporaneous value of another endogenous variable, Y_2 , the lagged value of the dependent variable, and some variable x that is exogenous to the model.

When actual values for Y_{2t} , Y_{1t-1} , and x_t are inserted into the RHS of the equation, an estimate, \hat{Y}_{1t} , of the value for Y_{1t} is generated.

In general \hat{Y}_{1t} will not be equal to the actual value, Y_{1t}^A for variable Y_1 during time period t .

The variable R_{1t} is called an "add factor". Observe that if the value of R_{1t} is set equal to $(Y_{1t}^A - \hat{Y}_{1t})$, then the RHS of the equation will generate a value that is exactly equal to Y_{1t}^A .

Each equation in the FOCUS model has an add factor. In preparing

the control solution every equation was first individually solved for each time period from 1971:1 to 1987:4 with its add factor set equal to zero. The results of this procedure were then used to re-calculate the add factors such that R_{it} was set equal to $(Y_{it}^A - \hat{Y}_{it})$ for all i and all t . With the add factors re-calibrated in this way, the model was re-solved in a simultaneous dynamic fashion over the same time interval. Because each equation now fitted the historical data perfectly, the resulting control solution exactly replicated history.

2. Generating the Alternative Solutions

The mechanics of generating the alternative solutions will be illustrated using the MURB program. Two things were done to the model to perform the simulations relating to MURB's. First, values for the variable, MURB, which appears as an (exogenous) independent variable in the multiple starts equation (Regression 3a in Table 7a of Appendix C), were set equal to zero over the entire simulation interval. The effect of this was to remove the incentive to build multiple units associated with MURB's as estimated in the housing sub-model.

Second, the add factors for federal and provincial personal income tax payments were increased by amounts equal to the estimated revenue losses attributable to MURB's since their initiation at the end of 1974. In effect, these alterations directly remove MURB's from the tax system.

The estimated values for these direct income tax effects are shown in Table 1, which also shows estimates of the direct costs of the other rental subsidy programs.

The FOCUS model was re-solved after making the indicated changes to

the MURB variable and the income tax add factors to generate the alternative solution associated with MURB's. Since MURB's did not exist prior to 1974:4, the solution interval for this alternative excludes the 1971:1 to 1974:3 sub-interval.

Similar procedures were used to obtain the other alternative solutions.

3. Policy Reaction Assumptions

Generating the counter-factual representations of macroeconomic performance depicted in the alternative solutions required that specific assumptions be made regarding the behaviour of government authorities. If we are to consider how the rental subsidy programs might have impacted upon the macro-economy, we must also conjecture how monetary and fiscal policies might have differed in their absence.

Ideally, we would like to keep monetary and fiscal policies unchanged as we move from the control solution to each alternative solution -- in order to identify the impacts due solely to the rental subsidy programs. In practice, it is not possible to keep all monetary and fiscal variables unchanged. For example, if tax rates are kept the same in all the solutions, then tax collections will differ between the control solution and the various alternatives. If tax collections are kept at the same values in all solutions, then implicit tax rates will differ amongst them.

In obtaining the alternative solutions the following policy assumptions were made:

- (i) Government expenditures on real goods and services were

maintained at historical values in all solutions. As a consequence, nominal expenditures were permitted to change in response to simulated changes in price levels. Since the price level impacts of the various rental subsidy programs turned out to be quite small, nominal government expenditures do not differ much across the various solutions.

- (ii) Tax rates were maintained at historical values in all solutions, so that tax collections vary across the alternatives in response to simulated changes in tax bases.
- (iii) The \$US/\$Canadian exchange rate was maintained at historical values in all solutions.
- (iv) The narrowly-defined money supply (M1) was maintained at historical values in all solutions, but the amount of domestic credit (i.e., the quantity of loanable funds) supplied by the Bank of Canada was permitted to vary in such a way as to keep the value of Canada's foreign indebtedness at historical values in all of the solutions.

Assumptions (i) - (iii) are more-or-less "standard" operating procedures for macro-econometric simulations designed to isolate the impacts of a specific event or combination of events. The constant M1 part of assumption (iv) is also "standard". The part of assumption (iv) relating to maintenance of constant foreign indebtedness was deemed

desirable here because of the nature of the events under consideration.

CMHC's rental subsidy programs resulted in the construction of substantial numbers of new multiple-unit buildings. Since multiple unit buildings have a high loan-to-value ratio, the subsidy programs undoubtedly resulted in significant increases in mortgage borrowing. In the absence of an expansion in domestic credit availability, this additional borrowing would be likely to displace other domestic borrowers and force a significant fraction of these competitors to move into international markets. Because foreign indebtedness was a concern of the Bank of Canada over much of the simulation interval, it is not unreasonable to suppose that the Bank might have altered the availability of domestic credit in response to CMHC's subsidy programs. Assumption (iv) is one way to allow for this accommodation. (It is by no means the only way to model this. For example, the foreign exchange rate could be permitted to adjust to simulated changes in international borrowing.)

4. Simulating Vacancy Rates in the Presence of Rent Controls

Estimates of the impacts of each of the various subsidy programs on the vacancy rate in multiple-unit rental apartment buildings appear in the final rows of Tables 2-5 and in the final row of Table 6 for the all-programs simulation. These estimates should be interpreted as showing by how much the various subsidy programs caused the economy-wide vacancy rate to change relative to what might have occurred in the programs' absence. Thus the value 1.1 (percent points), shown in Table 6 for 1984, estimates that the vacancy rate was 1.1 percentage points higher in 1984 than it might have been had none of the subsidy programs been put into

effect. The actual historical value for the economy-wide vacancy rate in 1984 was 2.2 percent. The all-programs simulation, therefore, suggests that the vacancy rate might have been only 1.1 percent if none of the subsidy programs had been put into effect after the end of 1970.

Examination of Tables 2-6 reveals that each of the subsidy programs has an estimated impact on the vacancy rate that is positive or zero in all years. The explanation for this is that each of the programs stimulated net new starts of multiple units, which in turn causes the stock of multiple units to have been higher in the presence of the programs than would have been the case in their absence. In a free-market setting, an increase in the stock supply of multiple units can be expected to lead to a short-run increase in vacancy rates. The demographic and economic factors that cause the stock demand for multiple units to increase in response to an increase in the stock supply operate with a lag.

The equilibrating mechanism at work in a free-market is described by the following.



This mechanism is incorporated in the rent and vacancy equation developed in the econometric model of the preceding Appendix. As was discussed in that Appendix, the existence of rent controls over part of the country during much of the simulation interval inhibits the market response of rents to imbalances between stock supplies and demands. Rent controls are likely to result in queues of would-be renters in areas of

the country in which the controls are binding. This is due to the fact that rents are artificially kept below market-clearing values. When this situation prevails, the measured value for the economy-wide vacancy rate is likely to provide a poor indication of market supply and demand conditions. More specifically, a particular value for the economy-wide vacancy rate is apt to understate the extent of excess demand that exists in regions with binding rent controls.

Where rent controls bind, an increase in the stock of multiple units is likely to give rise to an immediate increase in the number of occupied units, perhaps even on a one-for-one basis. This kind of response is something that no econometric model can adequately reflect. Data pertaining to queues of would-be renters do not exist and cannot, therefore, be used in estimating the behavioural parameters of a model.

This is not to say that our model of the residential housing sector fails to address some of the more important implications of rent controls. In particular, the estimated equation determining the stock demand for occupied multiple housing units shows that the existence of rent controls prevents renters from adjusting occupancy rates in response to changes in rents and real incomes as readily as would occur in a free market environment. There is clear evidence in our empirical work that rent controls lead to pent-up demands and also to a slowdown in the rate at which rents adjust to market conditions. What our estimated model is not able to accurately depict is how changes in the stock supply of multiple units provide immediate release of some of these pent-up demands. To our knowledge no existing model is capable of adequately representing this effect.

For these reasons, and also to enhance the intuitive appeal of our reported results, we have assumed that an increase in the stock supply of multiple units induces an immediate 75 percent offsetting increase in occupancies. This assumption is incorporated into the impact estimates shown for the vacancy rate in Tables 2-6, and has no implications for any of the other variables appearing in the simulation results.

B. The Simulation Results

Discussion of the simulation results, presented here in the form of impact estimates, will focus on the "all programs" simulation summarized in Table 6. The salient features of the results are summarized below, with the key findings identified in point form.

The impact estimates shown near the bottom of Table 6 for multiple starts are the results of most relevance here. These figures represent estimates of the net impacts of the combined rental subsidy programs on multiple starts after taking account of all feedbacks working through incomes, prices and stock-flow mechanisms. The figures reveal net impacts ranging from a high of +36.5 thousand starts in 1981 to a low of -3.0 thousand starts in 1987.

On a year-by-year basis the net impacts do not differ a great deal from the direct impact estimates derived from the single-equation simulation of the estimated equation for multiple starts reported near the end of Appendix C. A comparison of the results from Appendix C with those from Table 6 is presented in Table 7.

Column (3) of Table 7 shows the net direct impacts of the combined subsidy programs obtained from single-equation simulations. Column (5)

reports the net impacts from the full macroeconomic model simulation. The differences between these two estimates, shown in Column (4), represent the induced, or indirect, impacts attributable to macroeconomic feedbacks working through incomes, prices and lagged stocks (but not lagged starts, the effects of which appear in Column (3)).

- The induced impacts on multiple starts are uniformly quite small in magnitude and negative in sign.

This particular finding is a potentially valuable result; for it implies that macro-economic feedback effects are not particularly strong. Specific policies aimed at stimulating multiple starts via subsidies or tax incentive measures are apparently not diluted appreciably in their impacts by offsetting movements in incomes and/or prices.

The small magnitudes of the induced impacts on starts is something that would be difficult to predict in the absence of simulations with a macro-econometric model. The finding that the induced impacts are uniformly negative in sign is, on the other hand, an anticipated result -- and one predicted in the discussion of Appendix A.

The various rental subsidy programs were each temporary in duration. A temporary rental subsidy program operates very much like a temporary investment tax credit -- causing an acceleration in multiple starts that, in effect, borrows from the future. The stock of multiple units is stimulated to rise faster than otherwise during the program's existence. Once the program terminates, the high level of housing stock sets in motion factors that cause future starts to be lower than would have otherwise occurred. This depression in post-program starts constitute the negative induced impacts of the program. The forces that give rise to them are the lowered rents and increased vacancy rates that arise because of the initial acceleration in starts. These depressing forces will persist until the stock of houses is restored to its original "equilibrium" position.

Observe in Table 6 that the net impact of the subsidy programs on the multiple unit housing stock reaches a peak of 173 thousand units (or 4.7 percent of the existing stock) in 1986. The impact estimate for 1987 is lightly lower at 171.4 thousand units. If it were possible to extend the simulation exercises into 1988 and beyond, we could expect to see the impact estimate for the multiple-unit housing stock gradually approach zero as time passes.

How swiftly might this equilibration process proceed? An answer is suggested in Table 2, which reports impact estimates for the Limited Dividend program in isolation. The Limited Dividend program expired at the end of 1975, and so the simulation reported in Table 2 has a post-program sub-interval spanning 12 years (1976-87).

Observe from Table 2 that impact estimate for the stock of multiple units peaks at +20.4 thousand in 1978 and declines thereafter as a consequence of negative (induced) starts. However, by 1987 the impact estimate for the stock of multiple units is still +17.6 thousand -- a decline of only 2.8 thousand units from the peak. The return to "equilibrium" in the housing stock is clearly a slow process.

- The implication of the simulation results is that an increased stock of multiple units attributable to the rental subsidy programs is liable to persist for several decades into the future. The programs were temporary but their effects appear to be long-lasting.

Why is the equilibration process so slow to work after the programs' termination? There are two contributing factors. First, the net rate of removals from the existing stock of housing in Canada is quite low. Estimates described in Appendix C suggest that the rate of net removals from the existing stock due to demolition, fire, conversions, etc. is only marginally greater than zero in Canada for both single and

multiple units.

Second, the forces by which an increase in the stock of multiple units gives rise to a reduction in future starts are also slow moving. The mechanism by which this occurs in the housing model is the rent/vacancy nexus described in an earlier subsection. An increase in stock supply causes vacancy rates to rise which, in turn, puts downward pressures on rents. A decline in rents reduces the profitability of constructing new units and multiple starts decline.

The elasticity of multiple starts with respect to rents is of modest magnitude in the estimated housing model. In addition, rents are not very responsive to increases in the vacancy rate whenever rent controls are in effect in Ontario and/or British Columbia. Rent controls were in effect in these two provinces over most of the simulation interval. As a consequence, an increased stock of multiple units is not able to force market rents to decline by very much.

Table 6 does show a decline in rents as a consequence of an increased stock of multiple units, but the decline is small relative to what might be expected in the absence of rent controls.

- The presence of rent controls appears to be a major factor preventing rents from responding rapidly to increases in the stock supply of multiple units stimulated by the subsidy programs. The slow movement in rents prevents the induced impacts on new multiple starts from being the large negative magnitudes required to rapidly restore the stock supply to its equilibrium value.

To restate the point made earlier, the simulations results imply that the higher-than-otherwise stocks of multiple units attributable to the subsidy programs can be expected to persist for some time into the future. It is not possible to derive a precise estimate of how long it may take for the rent/vacancy equilibration process to restore the

housing stock to its natural equilibrium value. Among other factors, the future speed of the equilibration process will depend upon the future course of rent controls in Canada. Some idea of how rent controls affect this rate of adjustment may be learned from the impact estimates for rents in Table 6. The noticeable acceleration in the decline in rents shown in the table after 1984 is in large part explained by the removal of rent controls in British Columbia during that year.

Should rent controls remain in effect in Ontario and not be introduced elsewhere, a not unreasonable guess is that the equilibration process may be about half-way complete by the year 2000. In other words, we might expect the stock of multiple units to be about 86,500 thousand units ($-1/2$ of 173,000) higher in the year 2000 as a consequence of having had the subsidy programs in the 1970's and 1980's. This is, of course, a highly subjective estimate.

- The rental subsidy programs appear to have had little impact on either the price or the quantity of single-detached housing.

Our empirical investigation of Appendix C revealed no evidence of cross-substitution between demands for single and multiple units. It is not surprising, therefore, that the simulations show little impacts on the market for single-detached housing.

The results reported in Table 6 do indicate a small increase in single-detached prices, coupled with a slight decline in single starts in the latter part of the simulation interval. The former effect is due to a small increase in real disposable income attributable to the rental subsidy programs. Higher income increases the stock demand for single units and marginally increases selling prices. Single starts decline despite the rise in prices because the net economic expansion associated

with increased construction of multiple units also causes a slight increase in nominal interest rates on conventional mortgages. Higher interest rates outweigh higher selling prices in their effects on the profitability of new single starts.

The stimulative impacts of the subsidy programs on multiple starts are transmitted to other sectors of the economy. Higher starts in any quarter translate into increased outlays for real residential construction expenditures in that and the ensuing 4 or 5 quarters. Construction employment (not estimated separately) rises and the incomes generated by increased construction activity stimulate multiplied increases in consumption and imports. The multiplier effects stimulated by the subsidy programs reach all of the other sectors of the economy.

- The multiplier effects of the rental subsidy programs on real GDP, consumption, non-residential fixed investment, employment and the overall price level are uniformly small.

By increasing multiple starts during the time periods in which they are operative the various rental subsidy programs add positively to overall economic activity. The maximum impetus to real GDP is an additional \$695 million (in 1981 prices) occurring in 1976. This amount is quite naturally concentrated in real investment in residential construction but represents only 0.23 percent of the level of real GDP for that year. The subsidy programs exert large impacts on housing but these effects are small relative to aggregate total output.

The maximum expansion in employment is 9.2 thousand persons in 1977. This, too, is a small number in the overall scale of the economy and represents a reduction in the unemployment rate of less than one-half of one percent.

The aggregate output and employment impacts of the subsidy programs turn negative in sign in the latter part of the simulation interval after all of the programs have terminated. The magnitudes are never large.

The good news in these findings is that policies directed at the rental housing market can by and large be evaluated and implemented on their own merits. Policy makers need not be overly concerned that subsidies and tax incentives aimed at dealing with specific problems in rental housing will have potentially destabilizing impacts on overall macro-economic performance.

A final result of some interest has to do with the estimated impact effects on budget surpluses/deficits of the federal and provincial governments. The direct costs of the various programs collectively cause these budget balances to move toward deficits. It is worth noting that these direct costs extend in each case beyond the programs' respective dates of expiry.

The macroeconomic expansion which occurs while a program is operative stimulates an expansion in tax bases that allows some recapture of costs in the form of increased tax collections. The economic contraction that occurs after a program expires has the opposite effect.

The surplus/deficit figures reported in Table 6 show the net impacts of direct costs, less tax recapture on overall budgetary balances. For the all-programs simulation depicted in this table the expansionary phase for the macro-economy lasts longer than the contractionary, or post-program, phase. The figures, therefore, show more (positive) tax recapture than might be expected with a longer post-program simulation interval. Table 8 provides a summary for the all programs simulation.

- Over the long run it is reasonable to anticipate that the present value of government revenues foregone during the post-program phase will be approximately equal to the present value of additional government revenues gained prior to program expiry. The direct program costs should provide a reasonable accurate measure of the true net costs of the various programs. Because the subsidy programs exert only temporary (albeit, long-lived) impacts on housing starts and stocks, they cannot permanently alter real GDP and other macro-economic variables that underlie governments' real revenues and expenditures.

C. "High" and "Low" Alternatives

The simulation results reported in Tables 2-8 are based on Appendix C's point estimates of the responsiveness of multiple starts to the capitalized values of the subsidies inherent in the various rental subsidy programs. These point estimates are the values for the regression coefficients for the variables designated MURB and (LD+ARP+CRSP) in regression equation 3a, Table 7a, Appendix C.

Because these regression coefficients are point estimates and subject to sampling error, the impact estimates reported in this Appendix are point estimates of the macroeconomic impacts of the subsidy programs and are also subject to sampling error. Some idea of the statistical reliability of the impact estimates may be gained from a macroeconomic sensitivity analysis.

Table 9 reports the results of a macroeconomic simulation of all programs based on a variant of the multiple starts equation in which the coefficients on the MURB and (LD+ARP+CRSP) variables have been increased by one standard error, respectively, relative to the values in regression equation (3a). These results are labelled as our "High" estimates of the macroeconomic impacts relative to the point estimates reported in Table 6.

Table 10 reports the results of simulating all-programs based on a variant of the multiple starts equation in which the coefficient on the MURB variable has been reduced by one standard deviation (relative to its value in regression equation (3a)) and the coefficient on (LD+ARP+CRSP) has been set equal to zero. The point estimate of the coefficient for the latter variable is slightly less than one standard deviation above zero in regression equation (3a). Setting it equal to zero in this simulation effectively attributes zero net direct impacts on multiple starts to the LD, ARP and CRSP programs. The Table 10 impact estimates are labelled as "Low" estimates relative to the point estimates of Table 6.

The purpose of this exercise is to provide some evidence on the sensitivity of the macroeconomic impact estimates to variations in the key coefficients of the underlying model of the residential housing market. Comparison of the figures in Tables 6, 9 and 10 reveals that the simulation results span a wide range of impact estimates with the major differences concentrated, not surprisingly, in the variables relating to the sub-market for multiple housing units.

In a sense the "High" and "Low" alternatives define a confidence interval about the point estimates. However, it is not possible to attach a specific significance level to this interval. One reason is that the probabilities that the "true" coefficients for the MURB and (LD+ARP+CRSP) are both one standard deviation above or below the point estimates of regression equation (3a) are indeterminant. Our intuitive judgement is that a very high value should be assigned to the probability that the "true" impact estimates lie within the range spanned by the

"High" and "Low" simulations.

In spite of the broad range of impacts spanned by these two simulations, a couple of the inferences drawn from the point estimates remain valid.

These are:

- The induced (as opposed to the net direct) impacts of the subsidy programs on multiple starts and housing stock are small and slow-working.
- Relative to their impacts on the multiple housing market, the multiplier effects of the subsidy programs on real GDP, employment, price levels, etc. are small. This is true even in the "High" simulation in which the maximum impact on real GDP (occurring in 1976) represents less than one-half of one percent of the level of this variable.

D. Cyclical Implications

Each of the MURB, ARP and CRSP programs was initiated during a period of poor performance in multiple-unit construction activity. Indeed, poor performance in this area of economic activity was very probably an important factor in the policy decision that lead to the implementation of each of these various rental subsidy programs.

Our point estimates of the programs' net impacts show that each of the three programs moderated a downturn in new multiple-unit housing starts. The 1974-1975 downturn co-incident with the advent of MURB's and the ARP program was, however, mild relative to the prolonged recession in multiple-unit starts during the 1980's. These comments are supported by the figures shown in Table 11, which presents a decomposition of the net impacts of each of the subsidy programs.

The first column of Table 11 reports on an annual basis our point estimates for the levels of multiple starts that might have occurred from

1971-87 had none of the subsidy programs been in effect. These values are the solution values for the all-programs alternative solution used to determine the impact estimates reported in Table 6. The second through fifth columns of the table report estimates of the net impacts on multiple starts, decomposed by type of subsidy program. The figures relating to the LD program come directly from Table 2; those relating to MURB's come directly from Table 3; and so on. The all-program estimates may not exactly match the sums of the estimates for the individual programs due to non-linearities in the housing and FOCUS models. Observe, however, that the sums of the individual program estimates are very close to the all programs values in all years. Apparently model non-linearities are not very important in simulations of this scale in which the overall macro-economic impacts are small. (One implication of this is that it is possible to closely approximate the combined impacts of, say, MURB's and the ARP program by simply adding together the net impact estimates of Tables 3 and 4.)

The figures reported in Table 11 support the following:

- MURB's and the ARP program jointly moderated a cyclical downturn in multiple starts between 1975 and 1978. This downturn would have been considerably more severe in the absence of these two subsidy programs.
- The initial MURB program expired at the end of 1979 but was re-instituted from 1980:4 through 1981:4. The simulation evidence suggests that the latter time period might have seen a disastrous decline in multiple starts had MURB's not been re-instituted. Some 36.9 thousand multiple starts, or 41.5% of actual starts, are attributable to MURB's in 1981.
- The LD program was formulated to address long-term concerns. The impact of its expiry at the end of 1975 on new starts during 1976-79 was more than offset by the positive impacts of MURB's and the ARP program.

The summary statistics reported near the bottom of Table 11 for the counter-factual solution values and actual multiple starts may be used to provide a more formal statement of the implications of the subsidy program for the cyclical performance of starts. The summary statistics are based on the quarterly values for the two series reported annually in the table.

These statistics suggest that both the mean value and the standard deviation of multiple starts might have been reduced -- relative to what actually occurred -- had none of the subsidy programs been in effect over the 1971:1 to 1987:4 time period. Alternatively, the figures imply that the existence of the programs caused the mean and standard deviation of multiple starts to be higher than might otherwise have been the case. The combined programs contributed to an increase in the mean value of multiple starts (from 89.3 thousand to 99.5 thousand per quarter, seasonally adjusted at annual rates) by proportionately more than they caused the standard deviation of starts to rise (from 35.48 thousand to 36.76 thousand). Consequently, the coefficient of variation (standard deviation ÷ mean) is lower, at 0.37, for actual starts than for the counter-factual solution.

The coefficient of variation provides one descriptive measure of the variability, or cyclical instability of a time series with a non-zero mean value. Using this measure, the subsidy programs appear to have modestly reduced the cyclical instability of multiple-unit housing starts over the simulation interval.

Means, standard deviations and coefficients of variation for the counter-factual all-programs alternative solution versus actual data are

presented for other selected variables in Table 12. These measures are appropriate descriptive statistics only for non-trended, or mean-stationary, time series. Real business investment in residential construction, total business fixed investment and real GDP all exhibited positive exponential trend growth over the simulation interval. Descriptive statistics reported in Table 12 relate to percentage deviations from exponential trends for these three variables. The trends were estimated by ordinary least squares regressions of natural logarithms of actual values of these variables on linear time trends. In each case, the de-trended variable exhibits a zero mean value over the interval based on actual data. The coefficient of variation is undefined for a time series with zero mean and the standard deviation provides an appropriate measure of cyclical instability.

Other variables appearing in the table are multiple starts, an index of capacity utilization in the private sector of the economy, the economy-wide unemployment rate and the nominal interest rate on conventional 5-year mortgages. None of these remaining variables exhibited an exponential trend and so the summary statistics are based on unadjusted values.

The statistics reported in Table 12 reveal that the subsidy programs contributed to a modest reduction in the standard deviation of de-trended real business investment in residential construction. Finding that the subsidy programs appear to have made this variable slightly less cyclically volatile comes as no great surprise. Observe, however, that the magnitude of the effect is exceedingly small.

De-trended values for real business fixed investment (which includes

both machinery and equipment and non-residential construction) and real GDP have slightly larger standard deviations using actual data versus the counter-factual solution values. The suggestion here is that the subsidy programs caused these variables to be slightly more cyclically unstable than would have otherwise been the case. These findings are somewhat surprising but the magnitudes are so small as to be of little practical significance.

The index of capacity utilization and the unemployment rate are alternatives to real GDP as indicators of overall economic activity. The subsidy programs appear to have had no significant effects on the cyclical instability of either of these variables, based on a comparison of the values of their coefficients of variation between counter-factual and actual data. A similar inference applies to the subsidy programs' impacts on the cyclical instability of the nominal rate of interest on conventional mortgages.

A final bit of evidence regarding the cyclical implications of the subsidy programs is presented in Table 13. This table compares the pairwise correlation coefficients of some of the variables identified in Table 12 between the counter-factual all-programs alternative solution and actual historical data. Here, as in the last table, real business investment in residential construction and real GDP refer to percentage deviations from exponential trends.

It is interesting to observe that the correlation coefficient between multiple starts and de-trended real GDP is essentially zero in the counter-factual solution. The implication is that multiple starts and real GDP would not have shared a common cyclical component over the

1971:1 to 1987:4 time interval in the absence of the subsidy programs.

The correlation coefficient between multiple starts and de-trended real GDP using actual data is +0.17. That this is higher than with the counter-factual solution is a consequence of the result that the subsidy programs had positive net impacts on both starts and real GDP on average over the simulation interval. The inference is that starts were slightly more pro-cyclical with respect to real GDP than might otherwise have been the case. However, one must be cautious not to read too much into this. The inference should not be interpreted as meaning that the existence of the subsidy programs caused multiple starts to be more responsive to an external disturbance in real GDP (due to, say, a decline in consumer expenditures) than would have been the case in the absence of the programs.

Whether multiple starts were more or less responsive to disturbances in real GDP originating outside the housing market is a question not answered by the simulation experiments. The answer hinges on whether the subsidy programs altered the structural mechanism by which multiple starts respond to market variables. There is no basis for arguing that the subsidy programs would have any such effects. The programs were temporary by design and seen to be so by market participants. Further, none of the subsidy programs in any way interfered with the operation of market forces (unlike the imposition of rent controls). Our model of housing sub-market views the subsidies offered under the various programs as strictly additive to the market-determined variables that determine new starts. Consequently, one implication of this study is that the responsiveness of multiple starts to cyclical movements in real GDP is

completely unaffected by the existence or non-existence of rental subsidy programs of the sort under examination.

What, then, does one learn from the correlation coefficients appearing in Table 13? Simply, how history might have been altered had the subsidy programs not been put into effect. For example, multiple starts and the nominal interest rate on mortgages might have been considerably more negatively correlated in the absence of the subsidy programs than was actually the case. The ARP and MURB programs were each brought into effect at times when market interest rates were rising rapidly in Canada. The positive stimulus provided to new starts by these programs helped to offset the negative impacts of rising interest rates on starts.

Note that the implication in this is not that the existence of subsidy programs causes builders to be less concerned with interest rate movements. The correct inference is that the judicious use of discretionary policies can counteract the consequences of adverse movements in market variables and exert a stabilizing influence on multiple starts.

Table 1

Estimates of the Direct Program Costs of the Various
C.M.H.C. Rental Subsidy Programs

(NI&E Accounts (accrual) Basis, \$ Millions)

Year	Program							
	*Limited Dividend	**M.U.R.B.		+A.R.P.		++C.R.S.P.		TOTAL
	Federal	Fed.	Prov.	Fed.	Prov.	Fed.	Fed.	Prov.
1971	4.6	-	-	-	-	-	4.6	-
1972	6.5	-	-	-	-	-	6.5	-
1973	7.7	-	-	-	-	-	7.7	-
1974	8.9	-	-	-	-	-	8.9	-
1975	13.6	0.3	0.2	0.2	0.0	-	14.2	0.2
1976	13.8	1.9	1.0	2.6	0.5	-	18.3	1.5
1977	13.9	7.4	3.7	9.9	2.0	-	31.2	9.4
1978	14.0	16.5	8.2	17.7	3.5	-	48.2	11.7
1979	14.0	29.2	14.5	19.5	4.0	-	62.7	18.5
1980	14.0	39.9	20.4	25.0	5.0	-	78.9	25.4
1981	14.0	55.7	27.8	29.7	5.9	-	99.4	33.7
1982	14.0	69.4	34.7	32.3	6.4	1.2	117.4	41.1
1983	14.0	82.1	41.0	36.8	7.3	5.4	138.3	48.3
1984	14.0	93.7	46.9	46.4	9.3	15.1	169.2	56.2
1985	14.0	102.4	51.1	35.3	7.0	15.4	167.1	58.1
1986	14.0	101.3	50.8	47.9	9.6	23.2	186.4	60.4
1987	14.0	80.4	40.2	51.9	10.4	24.1	170.4	50.6

* Estimates of federal subsidies are derived from the Annual Non-Budgetary Funds authorized under LD, as recorded by Canadian Housing Statistics (CHS).

** The direct costs of M.U.R.B.'s are estimates of losses in personal income tax accruals to the federal and provincial governments associated with this program. The estimation technique is detailed in Appendix A.

+ Estimates of the direct costs of the Assisted Rental Program consist of federal subsidies to business and provincial subsidies to business resulting from "top ups" in Ontario and British Columbia (estimated at 20% of federal direct costs). The federal subsidies are estimated by the Annual Budgetary Expenditures under ARP (CHS).

++ Direct costs of CRSP are estimated by the Annual Budgetary Expenditures under CRSP (CHS).

Table 2

FOCUS MODEL - INSTITUTE FOR POLICY ANALYSIS
CMHC Housing Incentives - Limited Dividend Program May 9/89

	1971	1972	1973	1974	1975	1976	1977	1978	1979
Real Output and Components (Changes in \$ Mill)									
Real Gross Domestic Product	20	47	66	102	169	142	46	-4	-19
Consumption	6	21	38	54	74	93	64	13	-9
Consumption of Durables	1	4	6	8	13	11	0	-10	-13
Business Investment - Total	19	40	48	81	143	94	3	-17	-17
Residential Construction	18	36	42	75	135	88	12	1	-3
Non-Residential Construction	0	2	3	2	3	3	-2	-7	-6
Machinery and Equipment	0	2	4	4	4	2	-6	-11	-8
Exports	0	1	1	1	0	0	-3	-6	-8
Imports	2	13	21	33	51	56	19	-8	-17
Prices (Change in per cent)									
Implicit Deflator for GDP	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Consumer Price Index, All Items	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Consumer Price Index, Rent	0.0	0.0	-0.1	-0.1	-0.2	-0.2	-0.2	-0.3	-0.3
Selling Price - Single Houses	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0
Employment									
Employment (Change in per cent)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Employment (Change in '000)	0.3	0.6	0.9	1.2	1.9	1.8	0.0	-1.4	-2.1
Unemployment Rate (% Pts)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Interest Rates (Change in % Pts)									
90-day Paper Rate	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Conventional Mortgage Rate	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Government Deficits (Change in \$ Mill)									
Federal Surplus/Deficit (\$ Mill)	-3	-2	0	7	15	18	5	-12	-24
Provincial Surplus/Deficit (\$ Mill)	1	3	5	9	16	16	7	-2	-8
Housing Starts, Stocks and Vacancies									
Housing Starts - Singles ('000)	0.0	0.0	0.0	0.1	0.0	0.0	0.0	-0.1	-0.1
Housing Starts - Multiples ('000)	1.6	2.3	2.4	5.0	7.9	2.1	-0.2	-0.3	-0.3
Housing Stock - Singles ('000)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Housing Stock - Multiples ('000)	0.0	0.9	2.7	5.0	8.8	14.6	19.0	20.4	20.3
Vacancy Rate (% Pts)	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.2	0.1

FOCUS MODEL - INSTITUTE FOR POLICY ANALYSIS
CMHC Housing Incentives - Limited Dividend Program May 9/89

[illegible]

Table 3

FOCUS MODEL - INSTITUTE FOR POLICY ANALYSIS
CMHC Housing Incentives - Effect of MURB's

May 9/89

	1974	1975	1976	1977	1978	1979	1980	1981	1982
Real Output and Components (Changes in \$ Mill)									
Real Gross Domestic Product	10	193	372	435	432	341	187	532	526
Consumption	2	39	139	219	229	201	140	147	267
Consumption of Durables	1	14	28	33	26	13	-4	16	31
Business Investment - Total	13	210	352	357	342	245	112	598	492
Residential Construction	13	199	325	333	342	270	157	626	482
Non-Residential Construction	0	3	11	12	5	-4	-13	-11	4
Machinery and Equipment	0	8	17	12	-5	-21	-32	-16	6
Exports	0	0	1	1	1	1	-4	-7	-6
Imports	2	50	129	154	148	114	60	189	267
Prices (Change in per cent)									
Implicit Deflator for GDP	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Consumer Price Index, All Items	0.0	0.0	0.0	-0.1	-0.1	0.0	0.0	0.0	-0.1
Consumer Price Index, Rent	0.0	0.0	0.0	0.0	-0.1	-0.1	-0.1	-0.2	-0.2
Selling Price - Single Houses	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.0	0.1
Employment									
Employment (Change in per cent)	0.0	0.0	0.1	0.1	0.1	0.0	0.0	0.0	0.0
Employment (Change in '000)	0.1	2.2	4.9	6.1	5.6	3.4	0.2	2.8	4.8
Unemployment Rate (% Pts)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Interest Rates (Change in % Pts)									
90-day Paper Rate	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1
Conventional Mortgage Rate	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.2
Government Deficits (Change in \$ Mill)									
Federal Surplus/Deficit (\$ Mill)	1	29	69	95	99	77	36	102	67
Provincial Surplus/Deficit (\$ Mill)	1	17	38	49	44	24	-9	27	27
Housing Starts, Stocks and Vacancies									
Housing Starts - Singles ('000)	0.0	0.0	0.1	0.0	-0.1	-0.2	-0.4	-0.5	-0.6
Housing Starts - Multiples ('000)	1.9	13.9	17.4	16.0	16.4	10.5	5.9	36.9	10.1
Housing Stock - Singles ('000)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1
Housing Stock - Multiples ('000)	0.0	1.7	10.0	24.4	40.4	55.7	67.9	77.9	98.3
Vacancy Rate (% Pts)	0.0	0.0	0.1	0.1	0.3	0.4	0.4	0.5	0.6

Table 3 (Cont'd.)

FOCUS MODEL - INSTITUTE FOR POLICY ANALYSIS
CMHC Housing Incentives - Effect of MURB's

May 9/89

	1983	1984	1985	1986	1987
Real Output and Components (Changes in \$ Mill)					
Real Gross Domestic Product	148	-24	-120	-125	-69
Consumption	232	31	-107	-87	-22
Consumption of Durables	9	-25	-44	-35	-15
Business Investment - Total	28	-61	-81	-76	-1
Residential Construction	44	-7	-23	-36	9
Non-Residential Construction	-10	-28	-30	-23	-12
Machinery and Equipment	-6	-25	-29	-17	2
Exports	-10	-12	-20	-30	-38
Imports	93	-30	-100	-75	1
Prices (Change in per cent)					
Implicit Deflator for GDP	0.0	0.0	0.0	0.1	0.1
Consumer Price Index, All Items	0.0	0.0	0.0	0.0	0.0
Consumer Price Index, Rent	-0.3	-0.5	-1.1	-1.7	-2.2
Selling Price - Single Houses	0.0	0.0	0.0	0.1	0.3
Employment					
Employment (Change in per cent)	0.0	0.0	-0.1	-0.1	0.0
Employment (Change in '000)	0.2	-4.5	-7.3	-7.7	-5.8
Unemployment Rate (% Pts)	0.0	0.0	0.0	0.0	0.0
Interest Rates (Change in % Pts)					
90-day Paper Rate	0.0	0.0	0.0	0.0	0.0
Conventional Mortgage Rate	0.2	0.2	0.2	0.2	0.0
Government Deficits (Change in \$ Mill)					
Federal Surplus/Deficit (\$ Mill)	1	-40	-112	-139	-104
Provincial Surplus/Deficit (\$ Mill)	-54	-129	-169	-158	-120
Housing Starts, Stocks and Vacancies					
Housing Starts - Singles ('000)	-0.8	-0.9	-0.9	-0.8	0.2
Housing Starts - Multiples ('000)	-0.2	-0.9	-1.4	-2.0	-2.1
Housing Stock - Singles ('000)	-0.6	-1.2	-1.9	-2.6	-3.0
Housing Stock - Multiples ('000)	117.5	124.4	124.3	123.2	121.5
Vacancy Rate (% Pts)	0.8	0.8	0.8	0.8	0.8

Table 4

FOCUS MODEL - INSTITUTE FOR POLICY ANALYSIS
CMHC Housing Incentives - Effect of ARP Program May 9/89

	1975	1976	1977	1978	1979	1980	1981	1982	1983
Real Output and Components (Changes in \$ Mill)									
Real Gross Domestic Product	59	182	181	142	67	16	3	6	18
Consumption	11	58	100	92	49	9	-8	-3	17
Consumption of Durables	5	14	15	8	-3	-10	-10	-7	-1
Business Investment - Total	68	178	141	93	26	-10	-8	2	10
Residential Construction	65	163	126	92	38	6	2	1	3
Non-Residential Construction	1	5	6	2	-4	-6	-5	-2	1
Machinery and Equipment	2	9	8	-1	-9	-10	-5	2	6
Exports	0	1	1	1	2	2	2	3	5
Imports	14	61	67	48	12	-18	-20	-7	10
Prices (Change in per cent)									
Implicit Deflator for GDP	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Consumer Price Index, All Items	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Consumer Price Index, Rent	0.0	0.0	0.0	0.0	0.0	-0.1	-0.1	-0.1	-0.1
Selling Price - Single Houses	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Employment									
Employment (Change in per cent)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Employment (Change in '000)	0.6	2.2	2.5	1.8	0.2	-1.2	-1.6	-1.4	-0.9
Unemployment Rate (% Pts)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Interest Rates (Change in % Pts)									
90-day Paper Rate	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Conventional Mortgage Rate	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Government Deficits (Change in \$ Mill)									
Federal Surplus/Deficit (\$ Mill)	9	29	33	22	5	-21	-39	-51	-46
Provincial Surplus/Deficit (\$ Mill)	5	18	21	14	1	-12	-19	-24	-23
Housing Starts, Stocks and Vacancies									
Housing Starts - Singles ('000)	0.0	0.0	0.0	0.0	-0.1	-0.1	-0.1	-0.1	-0.1
Housing Starts - Multiples ('000)	5.8	8.4	5.6	3.7	0.7	-0.1	-0.1	-0.1	-0.1
Housing Stock - Singles ('000)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Housing Stock - Multiples ('000)	0.2	3.4	9.9	16.4	20.8	22.8	23.3	23.3	23.1
Vacancy Rate (% Pts)	0.0	0.0	0.1	0.1	0.2	0.2	0.2	0.2	0.2

Table 4 (Cont'd.)

FOCUS MODEL - INSTITUTE FOR POLICY ANALYSIS
CMHC Housing Incentives - Effect of ARP Program May 9/89

	1984	1985	1986	1987
Real Output and Components (Changes in \$ Mill)				
Real Gross Domestic Product	26	22	23	33
Consumption	28	25	22	25
Consumption of Durables	3	4	6	10
Business Investment - Total	16	15	11	14
Residential Construction	4	4	4	9
Non-Residential Construction	2	2	1	1
Machinery and Equipment	9	8	6	4
Exports	8	14	19	24
Imports	23	29	29	32
Prices (Change in per cent)				
Implicit Deflator for GDP	0.0	0.0	0.0	0.0
Consumer Price Index, All Items	0.0	0.0	0.0	0.0
Consumer Price Index, Rent	-0.1	-0.2	-0.3	-0.4
Selling Price - Single Houses	0.0	0.0	0.0	0.0
Employment				
Employment (Change in per cent)	0.0	0.0	0.0	0.0
Employment (Change in '000)	-0.5	-0.3	0.0	0.5
Unemployment Rate (% Pts)	0.0	0.0	0.0	0.0
Interest Rates (Change in % Pts)				
90-day Paper Rate	0.0	0.0	0.0	0.0
Conventional Mortgage Rate	0.0	0.0	0.0	0.0
Government Deficits (Change in \$ Mill)				
Federal Surplus/Deficit (\$ Mill)	-52	-46	-66	-72
Provincial Surplus/Deficit (\$ Mill)	-27	-29	-37	-38
Housing Starts, Stocks and Vacancies				
Housing Starts - Singles ('000)	-0.1	-0.1	0.0	0.1
Housing Starts - Multiples ('000)	-0.1	-0.2	-0.3	-0.3
Housing Stock - Singles ('000)	0.0	0.0	0.0	0.0
Housing Stock - Multiples ('000)	23.0	22.9	22.7	22.5
Vacancy Rate (% Pts)	0.2	0.2	0.1	0.1

Table 5

FOCUS MODEL - INSTITUTE FOR POLICY ANALYSIS
CMHC Housing Incentives - Effect of CRSP Program May 9/89

	1982	1983	1984	1985	1986	1987
Real Output and Components (Changes in \$ Mill)						
Real Gross Domestic Product	40	79	109	106	66	39
Consumption	12	28	49	58	37	6
Consumption of Durables	4	8	9	7	-2	-6
Business Investment - Total	34	76	92	65	6	-4
Residential Construction	32	71	85	61	10	6
Non-Residential Construction	1	2	3	3	0	-3
Machinery and Equipment	1	3	4	2	-4	-8
Exports	0	1	1	3	6	9
Imports	3	28	38	25	-18	-35
Prices (Change in per cent)						
Implicit Deflator for GDP	0.0	0.0	0.0	0.0	0.0	0.0
Consumer Price Index, All Items	0.0	0.0	0.0	0.0	0.0	0.0
Consumer Price Index, Rent	0.0	0.0	0.0	0.0	-0.1	-0.1
Selling Price - Single Houses	0.0	0.0	0.0	0.0	0.0	0.0
Employment						
Employment (Change in per cent)	0.0	0.0	0.0	0.0	0.0	0.0
Employment (Change in '000)	0.4	1.0	1.4	1.3	0.5	-0.2
Unemployment Rate (% Pts)	0.0	0.0	0.0	0.0	0.0	0.0
Interest Rates (Change in % Pts)						
90-day Paper Rate	0.0	0.0	0.0	0.0	0.0	0.0
Conventional Mortgage Rate	0.0	0.0	0.0	0.0	0.0	0.0
Government Deficits (Change in \$ Mill)						
Federal Surplus/Deficit (\$ Mill)	9	13	13	14	-9	-22
Provincial Surplus/Deficit (\$ Mill)	7	16	23	21	6	-5
Housing Starts, Stocks and Vacancies						
Housing Starts - Singles ('000)	0.0	0.0	0.0	0.0	0.0	0.0
Housing Starts - Multiples ('000)	2.1	3.2	3.4	1.6	0.0	-0.1
Housing Stock - Singles ('000)	0.0	0.0	0.0	0.0	0.0	0.0
Housing Stock - Multiples ('000)	0.1	1.2	3.7	6.8	9.0	9.9
Vacancy Rate (% Pts)	0.0	0.0	0.0	0.0	0.0	0.1

Table 6

FOCUS MODEL - INSTITUTE FOR POLICY ANALYSIS
CMHC Housing Incentives - Effect of All Programs -

May 9/89

	1971	1972	1973	1974	1975	1976	1977	1978	1979
Real Output and Components (Changes in \$ Mill)									
Real Gross Domestic Product	20	47	66	111	415	695	667	579	397
Consumption	6	21	38	55	118	285	388	345	254
Consumption of Durables	1	4	6	9	29	52	51	32	6
Business Investment - Total	19	40	48	94	420	623	499	417	252
Residential Construction	18	36	42	88	399	576	472	436	307
Non-Residential Construction	0	2	3	2	7	20	16	0	-13
Machinery and Equipment	0	2	4	4	14	26	11	-20	-42
Exports	0	1	1	1	1	3	1	1	0
Imports	2	13	21	35	114	240	240	192	116
Prices (Change in per cent)									
Implicit Deflator for GDP	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Consumer Price Index, All Items	0.0	0.0	0.0	0.0	-0.1	-0.1	-0.1	-0.1	-0.1
Consumer Price Index, Rent	0.0	0.0	-0.1	-0.1	-0.3	-0.3	-0.3	-0.3	-0.4
Selling Price - Single Houses	0.0	0.0	0.0	0.0	0.1	0.2	0.2	0.1	0.1
Employment									
Employment (Change in per cent)	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.0
Employment (Change in '000)	0.3	0.6	0.9	1.3	4.8	9.1	9.2	6.9	2.5
Unemployment Rate (% Pts)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Interest Rates (Change in % Pts)									
90-day Paper Rate	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Conventional Mortgage Rate	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1
Government Deficits (Change in \$ Mill)									
Federal Surplus/Deficit (\$ Mill)	-3	-2	0	9	51	114	132	110	60
Provincial Surplus/Deficit (\$ Mill)	1	3	5	10	37	70	76	55	18
Housing Starts, Stocks and Vacancies									
Housing Starts - Singles ('000)	0.0	0.0	0.0	0.1	0.1	0.2	0.0	-0.2	-0.4
Housing Starts - Multiples ('000)	1.6	2.3	2.4	6.9	27.6	27.8	21.4	19.8	10.8
Housing Stock - Singles ('000)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Housing Stock - Multiples ('000)	0.0	0.9	2.7	5.0	10.7	28.0	53.4	77.1	96.7
Vacancy Rate (% Pts)	0.0	0.0	0.0	0.0	0.1	0.2	0.4	0.5	0.6

Table 6 (Cont'd.)

FOCUS MODEL - INSTITUTE FOR POLICY ANALYSIS
CMHC Housing Incentives - Effect of All Programs -

May 9/89

	1980	1981	1982	1983	1984	1985	1986	1987
Real Output and Components (Changes in \$ Mill)								
Real Gross Domestic Product	195	535	572	251	110	-12	-69	-12
Consumption	156	157	299	307	136	-4	-11	42
Consumption of Durables	-16	8	31	18	-10	-31	-30	-10
Business Investment - Total	90	587	528	114	45	-9	-73	9
Residential Construction	161	626	515	116	79	35	-35	22
Non-Residential Construction	-23	-17	3	-6	-23	-26	-24	-15
Machinery and Equipment	-48	-22	10	5	-11	-18	-14	3
Exports	-6	-11	-11	-15	-17	-24	-33	-42
Imports	39	178	283	148	46	-33	-55	14
Prices (Change in per cent)								
Implicit Deflator for GDP	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1
Consumer Price Index, All Items	0.0	-0.1	-0.1	0.0	0.0	0.0	0.0	0.0
Consumer Price Index, Rent	-0.4	-0.5	-0.5	-0.6	-0.9	-1.7	-2.5	-3.3
Selling Price - Single Houses	0.0	0.1	0.2	0.2	0.1	0.2	0.2	0.5
Employment								
Employment (Change in per cent)	0.0	0.0	0.0	0.0	0.0	-0.1	-0.1	-0.1
Employment (Change in '000)	-2.0	0.6	3.4	0.1	-3.9	-7.0	-8.1	-6.8
Unemployment Rate (% Pts)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Interest Rates (Change in % Pts)								
90-day Paper Rate	0.0	0.1	0.1	0.0	0.0	0.0	0.0	0.0
Conventional Mortgage Rate	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.0
Government Deficits (Change in \$ Mill)								
Federal Surplus/Deficit (\$ Mill)	-17	26	-18	-64	-108	-182	-253	-233
Provincial Surplus/Deficit (\$ Mill)	-27	3	4	-63	-134	-176	-185	-147
Housing Starts, Stocks and Vacancies								
Housing Starts - Singles ('000)	-0.6	-0.7	-0.8	-1.0	-1.0	-1.0	-0.9	0.4
Housing Starts - Multiples ('000)	5.5	36.5	11.8	2.6	2.0	-0.5	-2.8	-3.0
Housing Stock - Singles ('000)	-0.1	-0.5	-1.1	-1.8	-2.6	-3.5	-4.3	-4.8
Housing Stock - Multiples ('000)	110.7	120.9	141.1	161.0	169.9	172.5	173.0	171.4
Vacancy Rate (% Pts)	0.8	0.8	0.9	1.0	1.1	1.1	1.1	1.0

Table 7

Decompositon of the Impacts of C.M.H.C. Rental Subsidies
on Multiple-Unit Housing Starts, All Programs

(000's of starts)

	(1)	(2)	(3)	(4)	(5)
Year	*Gross Numbers of Starts Receiving Subsidies	Direct Offsets	Net Direct Impacts (=col(1)+col(2))	Induced Impacts	Total Net Direct and Induced Impacts (=col(3)+col(4))
1971	11.5	-8.9	2.6	-1.0	1.6
1972	8.8	-6.3	2.5	-0.2	2.3
1973	4.5	-2.0	2.5	-0.1	2.4
1974	2.5	4.7	7.2	-0.3	6.9
1975	31.8	-3.5	28.3	-0.7	27.6
1976	60.4	-31.9	28.5	-0.7	27.8
1977	139.3	-117.2	22.1	-0.7	21.4
1978	98.3	-77.8	20.5	-0.7	19.8
1979	76.6	-65.0	11.6	-0.8	10.8
1980	0	6.4	6.4	-0.9	5.5
1981	61.5	-24.1	37.4	-0.9	36.5
1982	10.4	2.4	12.8	-1.0	11.8
1983	10.3	-6.3	4.0	-1.4	2.6
1984	3.5	0.1	3.6	-1.6	2.0
1985	0	1.7	1.7	-2.2	-0.5
1986	0	0.1	0.1	-2.9	-2.8
1987	0	0.0	0.0	-3.0	-3.0

* Figures for 1975-78 are computed as the sum of starts receiving subsidies under the ARP program and starts possessing M.U.R.B. certificates. Since almost all ARP units also possessed M.U.R.B. certificates, there is some double-counting over this interval.

Table 8

Program Costs: All Programs, Combined

(NI&E Accounts Basis, \$ Millions)

Year	<u>*Direct Program Costs</u>		<u>Induced Costs</u>		<u>**Net Direct Plus Induced Costs</u>	
	Federal	Provincial	Federal	Provincial	Federal	Provincial
1971	4.6	0	-1.6	-1.0	3.0	-1.0
1972	6.5	0	-4.5	-3.0	2.0	-3.0
1973	7.7	0	-7.7	-5.0	0.0	-5.0
1974	8.9	0	-17.9	-10.0	-9.0	-10.0
1975	14.1	0.2	-65.1	-37.2	-51.0	-37.0
1976	18.3	1.5	-132.3	-71.5	-114.0	-70.0
1977	31.2	9.4	-163.2	-85.4	-132.0	-76.0
1978	48.2	11.7	-158.2	-66.7	-110.0	-55.0
1979	62.7	18.5	-122.7	-36.5	-60.0	-18.0
1980	78.9	25.4	-61.9	1.6	17.0	27.0
1981	99.4	33.7	-125.4	-36.7	-26.0	-3.0
1982	117.4	41.1	-99.4	-45.1	18.0	-4.0
1983	138.3	48.3	-74.3	-14.7	64.0	63.0
1984	169.2	56.2	-61.2	77.8	108.0	134.0
1985	167.1	58.1	14.9	117.9	182.0	176.0
1986	186.4	60.4	67.6	124.6	253.0	185.0
1987	170.4	50.6	62.6	96.4	233.0	147.0

* The figures in the first two columns are estimates of Direct Program Costs from Table 1.

** Equal to the impact estimates from Table 6.

Table 9

FOCUS MODEL - INSTITUTE FOR POLICY ANALYSIS
CMHC Housing Incentives - Effect of All Programs - *High* May 9/89

	1971	1972	1973	1974	1975	1976	1977	1978	1979
Real Output and Components (Changes in \$ Mill)									
Real Gross Domestic Product	42	109	150	236	802	1308	1156	917	539
Consumption	9	41	77	103	215	521	685	551	337
Consumption of Durables	3	8	12	16	51	96	88	45	-9
Business Investment - Total	47	97	115	216	830	1193	875	677	358
Residential Construction	44	88	102	203	792	1107	835	732	479
Non-Residential Construction	1	4	6	5	14	38	27	-7	-33
Machinery and Equipment	1	5	8	8	24	47	13	-49	-89
Exports	0	1	0	0	-1	1	-4	-8	-10
Imports	9	29	46	75	220	461	433	316	152
Prices (Change in per cent)									
Implicit Deflator for GDP	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Consumer Price Index, All Items	0.0	0.0	0.0	0.0	-0.1	-0.2	-0.1	-0.1	-0.1
Consumer Price Index, Rent	0.0	0.0	-0.1	-0.4	-0.7	-0.7	-0.7	-0.8	-0.9
Selling Price - Single Houses	0.0	0.0	0.1	0.1	0.1	0.3	0.3	0.2	0.1
Employment									
Employment (Change in per cent)	0.0	0.0	0.0	0.0	0.1	0.2	0.2	0.1	0.0
Employment (Change in '000)	0.5	1.4	2.0	2.8	9.4	17.3	16.2	10.6	1.8
Unemployment Rate (% Pts)	0.0	0.0	0.0	0.0	-0.1	-0.1	-0.1	0.0	0.0
Interest Rates (Change in % Pts)									
90-day Paper Rate	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1
Conventional Mortgage Rate	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.2	0.2
Government Deficits (Change in \$ Mill)									
Federal Surplus/Deficit (\$ Mill)	-1	6	14	34	123	246	278	236	144
Provincial Surplus/Deficit (\$ Mill)	2	7	11	21	73	133	137	91	18
Housing Starts, Stocks and Vacancies									
Housing Starts - Singles ('000)	0.0	0.0	0.1	0.1	0.1	0.2	-0.1	-0.6	-1.0
Housing Starts - Multiples ('000)	4.1	5.8	5.9	15.3	54.9	52.1	37.4	32.9	16.5
Housing Stock - Singles ('000)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4
Housing Stock - Multiples ('000)	0.3	2.5	7.1	12.9	25.3	60.2	109.0	152.3	185.8
Vacancy Rate (% Pts)	0.0	0.0	0.0	0.1	0.2	0.4	0.8	1.1	1.3

Table 9 (Cont'd.)

FOCUS MODEL - INSTITUTE FOR POLICY ANALYSIS
CMHC Housing Incentives - Effect of All Programs - *High* May 9/89

	1980	1981	1982	1983	1984	1985	1986	1987
Real Output and Components (Changes in \$ Mill)								
Real Gross Domestic Product	181	674	756	323	122	-102	-244	-127
Consumption	143	120	339	385	142	-91	-136	-65
Consumption of Durables	-47	-9	32	19	-22	-58	-63	-33
Business Investment - Total	84	837	793	218	127	7	-163	-28
Residential Construction	227	919	781	221	178	81	-82	24
Non-Residential Construction	-48	-36	0	-12	-36	-44	-46	-35
Machinery and Equipment	-96	-46	12	10	-15	-31	-36	-17
Exports	-21	-30	-29	-35	-36	-45	-52	-59
Imports	9	217	400	232	94	-42	-125	-40
Prices (Change in per cent)								
Implicit Deflator for GDP	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Consumer Price Index, All Items	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.1
Consumer Price Index, Rent	-1.0	-1.1	-1.2	-1.3	-1.8	-3.2	-4.7	-6.1
Selling Price - Single Houses	0.1	0.1	0.3	0.3	0.3	0.4	0.4	0.9
Employment								
Employment (Change in per cent)	-0.1	0.0	0.0	0.0	-0.1	-0.1	-0.1	-0.1
Employment (Change in '000)	-6.3	-2.4	2.5	-1.5	-6.6	-11.2	-13.6	-11.3
Unemployment Rate (% Pts)	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.0
Interest Rates (Change in % Pts)								
90-day Paper Rate	0.0	0.1	0.1	0.1	0.0	0.0	0.0	0.0
Conventional Mortgage Rate	0.3	0.3	0.4	0.4	0.4	0.4	0.4	0.0
Government Deficits (Change in \$ Mill)								
Federal Surplus/Deficit (\$ Mill)	17	89	40	13	4	-105	-219	-184
Provincial Surplus/Deficit (\$ Mill)	-64	-29	-34	-125	-233	-313	-348	-301
Housing Starts, Stocks and Vacancies								
Housing Starts - Singles ('000)	-1.3	-1.5	-1.7	-1.9	-1.9	-1.9	-1.8	0.7
Housing Starts - Multiples ('000)	7.7	54.2	19.2	6.5	5.8	-0.1	-5.3	-5.7
Housing Stock - Singles ('000)	-1.3	-2.5	-3.9	-5.4	-7.1	-8.8	-10.5	-11.6
Housing Stock - Multiples ('000)	208.5	223.8	253.7	284.2	299.5	305.8	308.0	305.5
Vacancy Rate (% Pts)	1.4	1.5	1.7	1.9	2.0	2.0	2.0	1.9

Table 10

FOCUS MODEL - INSTITUTE FOR POLICY ANALYSIS
CMHC Housing Incentives - Effect of All Programs - *Low* May 9/89

	1971	1972	1973	1974	1975	1976	1977	1978	1979
Real Output and Components (Changes in \$ Mill)									
Real Gross Domestic Product	7	21	27	38	139	242	289	312	285
Consumption	9	24	22	25	50	107	161	182	183
Consumption of Durables	3	5	1	2	11	21	26	25	20
Business Investment - Total	1	2	3	13	113	187	193	193	153
Residential Construction	0	1	1	10	104	169	177	187	156
Non-Residential Construction	0	0	1	2	4	8	8	6	2
Machinery and Equipment	0	1	1	1	5	9	7	2	-4
Exports	0	0	2	4	5	8	12	16	17
Imports	2	5	0	2	26	68	82	83	73
Prices (Change in per cent)									
Implicit Deflator for GDP	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	-0.1	-0.1
Consumer Price Index, All Items	0.0	0.0	0.0	0.0	0.0	-0.1	-0.1	-0.1	-0.1
Consumer Price Index, Rent	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1
Selling Price - Single Houses	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1
Employment									
Employment (Change in per cent)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Employment (Change in '000)	-0.2	-0.4	-0.1	0.1	1.3	2.7	3.3	3.3	2.4
Unemployment Rate (% Pts)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Interest Rates (Change in % Pts)									
90-day Paper Rate	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Conventional Mortgage Rate	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Government Deficits (Change in \$ Mill)									
Federal Surplus/Deficit (\$ Mill)	-5	-7	-10	-12	-5	9	12	3	-16
Provincial Surplus/Deficit (\$ Mill)	0	1	1	1	9	20	24	21	13
Housing Starts, Stocks and Vacancies									
Housing Starts - Singles ('000)	0.0	0.0	0.0	0.1	0.1	0.2	0.2	0.2	0.2
Housing Starts - Multiples ('000)	0.0	0.0	0.0	1.0	7.0	8.8	8.1	8.3	5.4
Housing Stock - Singles ('000)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Housing Stock - Multiples ('000)	0.0	0.0	0.0	0.0	0.9	5.1	12.3	20.4	28.2
Vacancy Rate (% Pts)	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.2

Table 10 (Cont'd.)

FOCUS MODEL - INSTITUTE FOR POLICY ANALYSIS
CMHC Housing Incentives - Effect of All Programs - *Low* May 9/89

	1980	1981	1982	1983	1984	1985	1986	1987
Real Output and Components (Changes in \$ Mill)								
Real Gross Domestic Product	219	405	414	222	148	119	138	150
Consumption	163	180	253	234	139	88	105	134
Consumption of Durables	12	23	30	17	1	-7	2	16
Business Investment - Total	95	345	296	68	29	24	30	53
Residential Construction	104	345	277	59	35	29	24	32
Non-Residential Construction	-1	0	8	2	-6	-5	0	5
Machinery and Equipment	-8	1	11	6	0	0	7	16
Exports	16	15	14	11	13	12	9	5
Imports	54	127	171	87	27	2	7	45
Prices (Change in per cent)								
Implicit Deflator for GDP	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Consumer Price Index, All Items	-0.1	-0.1	-0.1	-0.1	-0.1	0.0	0.0	0.0
Consumer Price Index, Rent	-0.1	-0.1	-0.1	-0.2	-0.3	-0.5	-0.8	-1.1
Selling Price - Single Houses	0.1	0.1	0.2	0.1	0.1	0.1	0.1	0.2
Employment								
Employment (Change in per cent)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Employment (Change in '000)	1.0	2.2	3.2	1.0	-1.3	-2.5	-2.2	-1.2
Unemployment Rate (% Pts)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Interest Rates (Change in % Pts)								
90-day Paper Rate	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Conventional Mortgage Rate	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Government Deficits (Change in \$ Mill)								
Federal Surplus/Deficit (\$ Mill)	-52	-40	-71	-129	-197	-235	-274	-267
Provincial Surplus/Deficit (\$ Mill)	-1	21	31	-10	-48	-61	-55	-29
Housing Starts, Stocks and Vacancies								
Housing Starts - Singles ('000)	0.1	0.1	0.1	0.0	-0.1	-0.1	0.0	0.2
Housing Starts - Multiples ('000)	3.1	18.6	5.2	0.1	-0.3	-0.5	-0.8	-0.9
Housing Stock - Singles ('000)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Housing Stock - Multiples ('000)	34.4	39.5	49.9	59.7	63.3	63.5	63.1	62.5
Vacancy Rate (% Pts)	0.2	0.2	0.3	0.4	0.4	0.4	0.4	0.4

Table 11

Decomposition of Net Impacts on Multiple Starts

(000's of units)

Multiple-Unit Housing Starts

Year	Estimated Value In All Programs Alternative Solution	Net Impact Due to					Historical Value
		LD	MURB	ARP	CRSP	*All Programs	
1971	133.0	1.6	-	-	-	1.6	135.6
1972	132.0	2.3	-	-	-	2.3	134.3
1973	134.6	2.4	-	-	-	2.4	137.0
1974	93.1	5.0	1.9	-	-	6.9	100.0
1975	79.9	7.9	13.9	5.8	-	27.6	107.5
1976	111.1	2.1	17.4	8.4	-	27.8	138.9
1977	115.9	-0.2	16.0	5.6	-	21.4	137.3
1978	97.8	-0.3	16.4	3.7	-	19.8	117.6
1979	77.1	-0.3	10.5	0.7	-	10.8	87.9
1980	65.4	-0.3	5.9	-0.1	-	5.5	70.9
1981	52.4	-0.3	36.9	-0.1	-	36.5	88.9
1982	59.7	-0.3	10.1	-0.1	2.1	11.7	71.4
1983	57.7	-0.3	-0.2	-0.1	3.2	2.6	60.3
1984	49.2	-0.3	-0.9	-0.1	3.4	2.0	51.2
1985	67.7	-0.4	-1.4	-0.2	1.6	-0.5	67.2
1986	92.6	-0.5	-2.0	-0.3	0.0	-2.8	79.8
1987	108.8	-0.5	-2.1	-0.3	-0.1	-3.0	105.8
Mean Value	89.3						99.5
Standard Deviation	35.48						36.76
Coefficient of Variation	0.40						0.37

* The various program impacts may not sum to the all-programs impacts due to non-linearities in the models.

Table 12

Summary Measures of Cyclical Instability over 1971:1 to 1987:4

	Mean Value		Standard Deviation		Coefficient of Variation	
	All-Programs Solution	Actual Data	All-Programs Solution	Actual Data	All-Programs Solution	Actual Data
Multiple Starts	89.4	99.5	35.47	36.76	0.40	0.37
*Real Business Investment in Residential Construction	-1.20	0.00	10.06	9.96	-	-
*Real Business Fixed Investment	-0.38	0.00	7.16	7.33	-	-
*Real GDP	-0.08	0.00	3.15	3.19	-	-
Index of Capacity Utilization (=100 in 1971)	84.14	84.22	5.74	5.72	0.07	0.07
Unemployment Rate (per cent)	8.21	8.20	2.04	2.05	0.25	0.25
Nominal Interest Rate on Conventional Mortgages (per cent)	12.10	12.20	2.61	2.66	0.22	0.22

* Variable measured as percentage deviation from exponential trend.

Table 13

Selected Correlation Coefficients Based on the Counterfactual All-Programs
Solution Values Versus Actual Data 1971:1 to 1987:4

Variables	⁺ Correlation Coefficient Based on	
	The All-Programs Counter Factual Solution	Actual Data
Multiple Starts, Real GDP*	0.00	0.17
Multiple Starts, Capacity Utilization Rate	0.60	0.52
Multiple Starts, Unemployment Rate	-0.53	-0.59
Multiple Starts, Conventional Mortgage Rate	-0.59	-0.45
Real Business Investment in Residential Construction, Real GNP*	0.58	0.63
Real Business Investment in Residential Construction, Capacity Utilization Rate	0.55	0.51
Real Business Investment in Residential Construction, Unemployment Rate	-0.36	-0.37
Real Business Investment in Residential Construction, Conventional Mortgage Rate	-0.52	-0.46

* Variable measured as percentage deviation from exponential trend.

⁺ Figures in the first column show pair-wise correlation coefficients computed for quarterly values for the indicated variables from the all-programs alternative solution. Figures in the second column show pair-wise correlation coefficients computed from actual quarterly data.